

# E-GUIDE

# ALTERNATIVE TEACHING METHODOLOGIES IN MATH



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# Introduction

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Mathematics is a fundamental discipline that enhances individuals' problem-solving, critical thinking, and decision-making skills in daily life. However, for students aged 7–11 with learning difficulties (LD), mathematics learning can be challenging due to abstract concepts and traditional teaching methods. The *Empowering Mathematics Education for All* project aims to overcome these challenges and create an inclusive learning environment where every student can succeed in mathematics. This E-Guide provides teachers, parents, and educational stakeholders with practical, innovative, and student-centered strategies tailored for students with learning difficulties (Boaler, 2016).

The primary goal of the e-guide is to support mathematics learning for 7–11-year-old students with learning difficulties. Through methods such as visual representations, game-based learning, and scenario-based problem-solving, the guide aims to achieve the following objectives:

- enhance inclusivity by addressing diverse learning needs;
- make learning engaging through motivating and interactive activities;
- foster collaboration among teachers, parents, and stakeholders;
- provide applicable solutions for classroom and home environments.

The e-guide targets primary school teachers, parents, school administrators, and educational technologists. It consists of five main sections: theoretical framework, applied strategies, case studies, guide implementation and outcomes, and references. Each section combines evidence-based theory with practical applications to ensure accessibility and impact (Tomlinson, 2014).

We invite stakeholders to implement the E-Guide, share feedback, and collaborate to improve its content. Through these methods, we aim to create an environment where every child can succeed in mathematics.

## Theoretical Framework

The e-guide is grounded in contemporary educational theories emphasizing inclusive and differentiated instruction. According to Bruner (1966), learning is most effective when structured from concrete to abstract; this principle is fundamental for students with learning difficulties. Boaler (2016) advocates for a “mathematical mindset” approach that encourages flexible and creative problem-solving, boosting students' confidence. These theories shape the E-Guide's visual, kinesthetic, and auditory strategies tailored to diverse learning needs.

Tomlinson's (2014) differentiated instruction approach is another cornerstone, ensuring content, processes, and outcomes are adapted to students' readiness levels, interests, and learning profiles. For students with learning difficulties, such adaptations are critical to overcoming barriers like abstract reasoning or sustained attention (Van

Garderen, 2006). Additionally, the e-guide draws on Polya’s (1957) problem-solving framework, which emphasizes breaking complex tasks into manageable steps, particularly effective for LD students.

Table 1, based on educational research (Van Gardener, 2006), illustrates common challenges faced by students with learning difficulties in mathematics.

*Table 1. Common Challenges in Mathematics for Students with Learning Difficulties*

No.	Challenge	Description
1.	Abstract reasoning	Difficulty visualizing fractions or algebraic concepts
2.	Attention span	Difficulty focusing during multi-step problem-solving
3.	Memory retention	Forgetting procedural steps in operations like division

The theoretical framework ensures the e-guide’s strategies are evidence-based and tailored to LD students’ needs, promoting both academic success and positive attitudes toward mathematics.

**Applied strategies and guide:** this section offers teachers and parents practical strategies to enhance mathematics education for students with learning difficulties. Three key strategies are detailed: visual representations, prior knowledge activation, and explicit instruction method.

**Visual representations:** visual tools make abstract concepts concrete, facilitating understanding for LD students (Bruner, 1966). Teachers can use manipulatives like blocks or diagrams to demonstrate operations.

Implementation steps:

- 1) Students bring everyday objects (e.g., fruit pictures, beads) to class.
- 2) Operations (e.g.,  $3 + 2$ ) are demonstrated using these objects.
- 3) Students create their own visual models.

*Example:* To teach addition, draw 3 apples and 2 apples, combining them to show 5. Students replicate this with their objects. Classroom trials showed improved accuracy in basic operations.

Innovative suggestion: create interactive diagrams using digital tools like Canva, or organize a “Math Art Gallery” for students to display their models.

**Prior knowledge activation:** connecting new concepts to students’ existing knowledge facilitates learning (Hattie, 2012).

Implementation steps:

- 1) Ask students what they know about a topic (e.g., “Where do you use multiplication?”).
- 2) Create a mind map linking prior knowledge to new concepts.
- 3) Reinforce connections with real-life examples.

*Example:* For multiplication, ask, “If one orange costs 4 TL, how much do 3 oranges cost?” Draw 3 groups of 4 oranges to show 12. This approach links abstract operations to familiar contexts.

Innovative suggestion: students collect real-life math examples (e.g., grocery receipts) or enact a “Math Market” scenario.

**Explicit instruction method:** breaking complex tasks into smaller steps reduces cognitive load for LD students (Polya, 1957).

Implementation steps:

- 1) Write the problem and break it into steps.
- 2) Explain each step verbally and visually.
- 3) Students apply each step independently.

*Example:* To simplify  $18/24$ :

- Find the common divisor (6).
- Divide the numerator ( $18 \div 6 = 3$ ).
- Divide the denominator ( $24 \div 6 = 4$ ).
- Write the result ( $3/4$ ).

Students using this method showed increased proficiency in fraction operations.

Innovative suggestion: create “Step Cards” for correct sequencing or use apps like GeoGebra for animated step-by-step solutions.

These strategies are adaptable to diverse learning needs, ensuring inclusivity and engagement.

**Case studies and practical examples:** this section provides real-world examples from classroom and home settings addressing challenges faced by students with learning difficulties.

**Case Study 1: Fractions with Visual Representations**

*Observation:* a student incorrectly adds  $1/2 + 1/4$  as  $2/6$ , struggling to visualize fractions.

*Goal:* Strengthen fraction understanding with concrete tools.

*Activity:* the “Lego and Paper Circles Workshop” involves modeling fractions with colorful Legos and paper circles. Students combine  $1/2$  and  $1/4$  to see  $3/4$ .

*Home activity:* parents practice fractions using kitchen tools (e.g., measuring cups).

*Outcome:* test scores improved from 45% to 80%, reflecting stronger conceptual understanding.

**Case Study 2: Problem-Solving with Prior Knowledge**

*Observation:* a student confuses the order of operations in multi-step problems.

*Goal:* teach correct sequencing using real-life contexts.

*Activity:* the “Shopping Scenario” involves calculating costs (e.g., 3 apples at 5 TL each, with a 1 TL discount).

*Home activity:* parents simulate a market game at home.

*Outcome:* accuracy in operation sequencing increased from 40% to 80%.

These examples demonstrate how the e-guide’s strategies address LD challenges and support academic and emotional growth.

**Implementation and outcomes of the e-guide:** this section explains how to effectively implement the e-guide, providing guidance for teachers and parents.

*Teachers:*

- Integrate strategies into weekly lesson plans (e.g., game-based activities).
- Use digital tools like Khan Academy for interactive exercises.
- Collect student feedback to assess impact.

*Parents:*

- Implement home-based activities (e.g., fractions in the kitchen) with simple materials.

- Collaborate with teachers to address challenges.
- Reward children’s efforts to sustain motivation.

*Implementation process:*

- 1) Organize training workshops for stakeholders.
- 2) Apply strategies in pilot classrooms and homes.
- 3) Evaluate outcomes using pre/post-tests and feedback.

*Expected outcomes:*

- Improved student performance and confidence.
- Enhanced teacher effectiveness and parent engagement.
- Stronger school-family partnerships.

*Table 2. Stakeholder feedback on the effectiveness of the e-guide*

No.	Statement	Agree (Responses)	Disagree (Responses)
1.	The e-guide increased student engagement.	40	5
2.	The activities were easy to implement.	38	7

*Source: Pilot study data.*

Stakeholders are encouraged to share feedback via the project website to improve the e-guide and ensure it meets diverse educational needs.

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## Sources and Literature

1. Boaler, J. (2016). *Mathematical Mindsets*. San Francisco: Jossey-Bass.
2. Bruner, J. S. (1966). *Toward a Theory of Instruction*. Cambridge, MA: Harvard University Press.
3. Hattie, J. (2012). *Visible Learning for Teachers*. London: Routledge.
4. Polya, G. (1957). *How to Solve It*. Princeton, NJ: Princeton University Press.
5. Tomlinson, C. A. (2014). *The Differentiated Classroom*. Alexandria, VA: ASCD.
6. Van Garderen, D. (2006). Spatial visualization and mathematical problem solving. *Journal of Learning Disabilities*, 39(6), 496–506.

# Chapter 1

## What Are Alternative Teaching Methodologies?

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Developed by: **LSpecPA - LATVIA**

**A method is a system of didactic cooperation techniques between the teacher and the learner, through which students acquire new knowledge, skills, and abilities while also developing their cognitive abilities (Dictionary of Pedagogical Terms, 2000).**

**Traditional teaching methods** are instructional approaches that have been popular for centuries and are often described as passive forms of learning. These methods typically rely on the teacher's authority and presentation, while the student actively receives the information.

**The main traditional teaching methods include:**

- lectures (theoretical instruction);
- learning from books and other resources;
- classic tests and exams;
- teacher authority and discipline.

**When choosing a teaching method, several factors must be considered:**

- the regularities of the learning process and the resulting didactic principles;
- general and specific learning goals and objectives for the given stage of education;
- the specifics of the subject content to be acquired;
- the learning capabilities of the students;
- the particular external conditions of the learning process;
- the specific educational environment: the condition of the school's material resources, access to information technologies (e.g., internet availability), and the time allocated for learning;
- the teacher's own ability to manage the learning process (Pedagogs.lv, n.d.).

**Traditional methods primarily focus on the transmission of information and the students' ability to absorb and memorize that information.**

In recent years, they have been criticized for often failing to provide sufficient opportunities for students to develop creativity, critical thinking, or problem-solving skills - abilities that are essential for mastering mathematics. Therefore, modern pedagogy increasingly turns to alternative teaching methods, especially when working with students who have diverse learning difficulties.

**Alternative teaching methods are pedagogical approaches that differ from the traditional education system, offering new ways to teach and learn mathematics.**

These methods often emphasize creativity, individualized approaches, hands-on experiences, and adapting the learning process to each student's needs. They may include projects, group work, art, games, exploration, and the use of information technologies.

The goal of alternative teaching methods is to promote students' active participation in the learning process, develop critical thinking, emotional intelligence, and social skills - thereby creating a more dynamic and motivating learning environment.

Popular alternative methods include the Montessori method, the Reggio Emilia approach, Waldorf education, Nature Aesthetics, the Barboleta method, and others.

When learning mathematics, alternative methods are especially suitable for students with diverse learning styles or those who struggle in traditional school environments.

**Alternative teaching methods:**

- develop creativity and the ability to solve problems independently;
- promote active learning and student engagement rather than mere repetition of knowledge;

- emphasize the development of each student's personality and respect for their individual needs;
- offer a calmer and less stressful learning environment compared to traditional school education;
- allow students to discover their interests and engage in diverse activities;
- focus on learning through experience and exploration rather than teacher-led knowledge transmission;
- provide greater flexibility and autonomy in learning, allowing students to work at their own pace;
- offer practical knowledge and skills that are useful in everyday life, such as problem-solving, decision-making, organization, and communication.

Overall, **alternative teaching methods** help to organize and implement a learning process that is more student-centered, taking into account each learner's needs, interests, and potential. Each method is used in the interaction between teacher and student (Montessori, M., 2019).

Alternative methods promote the growth of disciplined, self-motivated students across all areas of development - cognitive, emotional, social, and physical. Students work independently or in groups using specially designed materials; they engage actively while maintaining mutual respect.

These methods often offer creative and innovative solutions to foster understanding and ensure that the learning process is tailored to the diverse needs of students.

#### **Core principles that characterize alternative teaching methods include:**

- **Student-centered approach** – focuses on students' needs, interests, and learning styles. These methods promote an individualized approach where students are active participants in the learning process.
- **Creativity and freedom** – includes creative elements, allowing students to express themselves artistically, scientifically, or in other ways that foster imagination and problem-solving skills.
- **Development of critical thinking** – encourages students to analyze, evaluate, and think independently. The emphasis is on understanding and applying knowledge, rather than rote memorization.
- **Experience-based learning** – practical experience and real-life examples are central to alternative methods. Students gain knowledge through direct engagement, discovery, and experimentation.
- **Education as a way of life** – emphasizes lifelong learning, not just knowledge gained at school, but also as a process of personal growth and improving life quality.
- **Social and emotional development** – recognizes and supports students' social and emotional growth, fostering collaboration, communication, and empathy.
- **Individual and group learning approaches** – acknowledges that every student learns differently, encouraging diverse approaches such as group work, project-based learning, games, and cooperative strategies.
- **Holistic education** – promotes a holistic approach that nurtures intellectual, physical, emotional, and spiritual development.

- **Openness and flexibility** – views the learning process as dynamic and adaptable, where both teachers and students are open to modifying methods to achieve optimal learning outcomes.

Each alternative teaching method offers diverse approaches and tools to support the development of students as well-rounded, creative individuals and to make their learning more meaningful.

At the core of alternative teaching methods are elements such as the **visualization of learning materials**, the **provision of clear instructions** that support task completion, and the **solving of real-life situations**, which enables students to successfully transfer these skills to similar situations in the future.

**Visual representations** – visual representations include various forms such as images, diagrams, graphs, maps, or other visual elements used to display data, ideas, or information. They help students understand complex content more easily by presenting it in a graphic format instead of relying solely on text. Visual representations are an essential and integral component of teaching mathematics, particularly beneficial for students with learning difficulties during the stages of concept comprehension and acquisition.

**Instructions** – instructions need to be clear, structured, and motivating. When providing information, it's important to follow these principles:

- **Use of clear and understandable language** – ensures that students grasp the task.
- **Step-by-step principle** – break the task into smaller, more manageable steps.
- **Demonstrate how to solve math problems** by using examples and showing what to do if confusion arises.
- **Avoid over-assisting** – refrain from doing the task for the student out of impatience if they are capable of succeeding independently with time and minimal support.

Understanding instructions is a crucial prerequisite for developing mathematical skills in students with learning difficulties.

**Stimulating environment** – a stimulating environment encourages the student's curiosity and desire to explore the world around them. Its goal is to foster independence and self-directed learning. A stimulating environment is a key factor in supporting the development of the student as an independent and self-reliant learner. The acquisition of mathematical knowledge plays a vital role in every student's development, as it is closely linked to key cognitive processes such as perception, thinking, memory, sensations, and mental imagery - all of which interact to enhance sensory abilities.

Alternative teaching methods enrich both the content and the form of the subject. Their application supports the development of core mathematical skills and fosters a deeper understanding of mathematical concepts. These methods encourage learning through hands-on activities and visual representations, making the learning process more engaging and enjoyable. They promote student development and lead to long-lasting knowledge and skill acquisition. Alternative approaches teach students *how* to think - not *what* to think - and help them discover multiple paths to solving problems.

*Exercise (Grade 2)*  **Task instruction: "Complete! Solve using addition and multiplication!"**

3. Papildini! Risini ar saskaitīšanu un reizināšanu!

a)  $5 + 5 + 5 + 5 = \dots$  jeb  $4 \cdot \dots$

d)  $6 \cdot 3 = \dots$  jeb  $3 + 3 \dots$

b)  $4 + 4 + 4 + 4 + 4 = \dots$  jeb  $5 \cdot \dots$

e)  $2 \cdot 7 = \dots$  jeb  $7 \dots$

c)  $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = \dots$  jeb  $\dots$

f)  $10 \cdot 4 = \dots$  jeb  $\dots$

Students are asked to:

- 1) **Look at a repeated addition sentence**, like  $5 + 5 + 5$ .
- 2) **Write the total.**
- 3) **Translate it into a multiplication sentence**, showing how many times the same number was added.

**Examples explained:**

A.  $5 + 5 + 5 = 15 \rightarrow$  That's adding 5 **three times**  $\rightarrow$  So:  $3 \times 5 = 15$  or, as written in Latvian structure:  $4 \cdot \dots$  means figuring out how many groups of a number there are. (Actually here it should be  $3 \cdot 5 = 15$ , so the original might have a typo.)

B.  $4 + 4 + 4 + 4 + 4 = 20 \rightarrow$  5 times 4  $\rightarrow 5 \times 4 = 20$

C. Eight 2s  $\rightarrow 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16 \rightarrow 8 \times 2 = 16$

D. The multiplication is already given:  $6 \times 3 = 18 \rightarrow$  Match it with addition  $\rightarrow 3 + 3 + 3 + 3 + 3 + 3 = 18$

E.  $2 \times 7 = 14 \rightarrow$  Repeat 7 two times  $\rightarrow 7 + 7 = 14$

F.  $10 \times 4 = 40 \rightarrow$  So  $4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 40$

**Learning objective:**

This exercise helps students:

- Recognize that **multiplication is repeated addition**
- Understand the **structure** of a multiplication expression:
  - $\rightarrow$  Number of groups  $\times$  Size of each group

Use available natural materials (a technique from the Nature Aesthetics method) – acorns, stones, chestnuts – to support understanding by replacing repeated addition with multiplication and writing the corresponding operations.

	
	
	
	
	$4 \times 5 = 20$

$5 \times 4 = 20$

The teacher's assessment activity focuses on evaluating both the learning process and the outcomes achieved by students. The goal of assessment is to guide students toward higher achievement and to encourage self-evaluation by:

- assessing students' achievements in acquiring knowledge, developing skills, forming values, and supporting personal growth;
- encouraging and supporting students' self-directed efforts toward better results;
- stimulating students' ability to engage in self-assessment (Beļickis, I., 2000).

**Alternative assessment** is an approach used to evaluate students' knowledge, skills, and development outside of traditional structured tests and formal examinations.

It is often used in education and professional development to provide insight into an individual's progress through everyday activities, experience, and observations.

The main feature of alternative assessment is its **flexibility and adaptability** to the individual student, their abilities, learning difficulties, and circumstances - with a focus on practical outcomes and student growth.

#### Examples of alternative assessment:

- **Portfolios** – students compile various works, projects, or assignments that reflect their progress and achievements, allowing teachers to evaluate development over an extended learning period.
- **Self-assessment** – students assess their own skills and knowledge, reflecting on how well they have met learning goals or improved within a subject.
- **Group work and collaboration** – students evaluate each other's contributions and performance in group tasks or projects, focusing on collaboration and teamwork skills.
- **Learning journals** – students write about their learning experiences, reflecting on what they have learned and how it can be applied in practice.
- **Observation and feedback** – teachers provide regular feedback on students' progress or behavior without using formal grades.

#### Advantages of alternative assessment:

- It allows greater focus on the student's growth and development rather than just final outcomes.
- The assessment can be more personalized and tailored to specific needs and goals.
- It encourages active engagement in the learning process instead of focusing solely on end results.

#### Challenges of alternative assessment:

- Ensuring objectivity and consistency can be difficult, as the method is less structured and often based on subjective observations.
- It can be more time- and resource-intensive due to the need for regular monitoring and feedback.

**In summary**, alternative assessment is a powerful tool for promoting learning and development, providing an opportunity to evaluate individual skills and knowledge in a more diverse and meaningful way.

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## Sources and Literature

1. Beļickis, I. (2000). *Vērtīborientēta mācību stunda*. Rīga: RaKa.
2. Pedagoģijas terminu skaidrojošā vārdnīca, 2000
3. <https://pedagogs.lv/2018/02/21/macibu-metozu-organizacijas-formu-un-lidzeklu-izv-ele-kura-ir-kura-ir-dazadi-macibu-materiala-apguves-limeni/>
4. Montessori, M. (2019) . *Absorbējošais prāts*. Jāņa Rozes apgāds

# Chapter 2

## Understanding Learning Difficulties In Mathematics

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Developed by: **DABG - BULGARIA**

Difficulties in learning mathematics are associated with the term “dyscalculia”. This term is used to describe specific difficulties in learning mathematics. It affects the ability to acquire mathematical skills and includes all types of mathematics problems ranging from inability to understand numbers to inability to apply mathematical principles while solving problems.

“**Dyscalculia**” comes from Greek and Latin which means “counting badly”. The prefix “dys” comes from Greek and means “badly”. “Calculia” comes from the Latin word “calculus”, which means “pebble” or one of the counters on an abacus. In the specialized literature you may also meet the term “Numlexia” which is a synonym for dyscalculia.”

**The list of difficulties dyscalculic students encounter is quite long:**

- difficulties in counting (forwards and backwards, counting in groups);
- a very poor sense of number magnitude;
- difficulty in identifying small/big, long/short, to measure and to compare;
- difficulty with conceptualisation time and learning to read analogue clocks;
- difficulty in the sequence of numbers, or transposing them when repeated, such as turning 89 into 98;
- difficulty in learning and remembering arithmetic “facts” (like multiplication tables);
- difficulty in performing arithmetic operations; inability to do it mentally;
- confusion in the mathematical symbols: +, -, ÷, ×, <, >;
- difficulty in understanding and implementing mathematical concepts, rules, formulae, and sequences.

A dyscalculic student can have difficulties in one or more of those categories. Some students are good at understanding numbers and have an excellent memory, but they have trouble with mathematical reasoning. Some, on the contrary, have a hard time understanding the numbers, but after spending a lot of time learning basic concepts, they can overcome most of their problems thanks to their other strengths. That is why understanding the nature of individual problems is a very important step in dealing with dyscalculia.

Dyscalculics continue to add on their fingers while peers have moved to more efficient methods, and prefer to use a calculator for even the simplest arithmetic. dyscalculics cope with mathematics by memorising rules but have no real understanding of what they are doing, so easily forget or misuse the correct procedure.

Dyscalculia, which is less studied compared to dyslexia, affects children and adults. According to many studies (Butterworth, 2010; Butterworth, Varma & Laurillard D, 2011; Kucian & von Aster, 2015), it occurs in 3 to 7% of the global population. It's important to note that there is no medical treatment for dyscalculia, which implies that the condition accompanies the individual throughout their life. Dyscalculia can be accompanied by other learning difficulties such as dyslexia, dyspraxia, and hyperactivity, making it even more challenging to diagnose.

It's important to note that **dyscalculia is not only about “not being good at math”**. It affects many aspects of the everyday lives of children and adults with this condition, not just their math skills. These difficulties may manifest in unexpected ways that may not be immediately recognised as related to math. At the same time, we need to be aware that **not**

**all math problems, even serious ones, are due to dyscalculia.** Difficulties in Math can be due to dyslexia, visual or auditory processing problems, ADHD or some other factors.

Children with math difficulties tend to avoid participating in games that require counting or calculating points because they struggle to keep track of the score, determine who is in the lead, and judge which move would earn them more points.

People with dyscalculia, including adults, often struggle with punctuality, being consistently late or arriving too early due to poor time orientation, which is a common challenge associated with the condition. Difficulties can also extend to spatial orientation, map reading, and following instructions.

### **There are several factors that cause problems in learning mathematics:**

- **Incomplete mastery of number facts** – this is the foundation for basic computation, since it teaches students to think mathematically.
- **Difficulty transferring knowledge and making connections** - students find it difficult to connect the abstract or conceptual objects of math with reality.
- **Incomplete understanding of the language of math** - some students have difficulty in comprehending verbal directions or understanding written explanation which makes solving word problems difficult for them.
- **Visual, spatial and perceptual difficulties** - students with deficits in visual perceptions and spatial orientation are very likely to experience difficulties with geometry.
- **Dyslexia (reading difficulties)** – due to comorbid dyslexia students may have difficulties with reading numbers; reading and comprehending the instructions and word problems; disposition of digits in numbers, etc.
- **Difficulties in understanding concepts and symbols** – this causes difficulties in understanding mathematical symbols, concepts of weight, space, direction and time, concepts of numbers, quantity of measures and concept of amount.
- **Difficulties with sequence and mathematical facts** - students have problems with arranging numbers, counting forward and backwards, comparison of quantities, the order of arithmetic operations, etc.
- **Difficulties in complex thinking and flexibility** – they cause problems in choosing the right strategy for solving math problems, especially when they require multiple steps.

Many teachers have encountered students with dyscalculia, so it's crucial to not only recognise it but also suggest appropriate strategies for dealing with it. The severity of symptoms of dyscalculia can vary, and coping strategies also differ. Therefore, the main task for teachers is to select the right strategy and adapt it to each student's individual needs. Because dyscalculia can manifest in different ways, a flexible approach is necessary. This is only possible when teachers have a variety of strategies at their disposal and can choose the most suitable one.

### **Where to start?**

1) **Gather as much information about the student as possible** – from their parents, from other teachers, from specialists working with them in and out of school, and also from the student: what he finds difficult (regarding learning in general and Maths in particular); can he work independently or needs help; how much time it takes him to complete tasks and if he tends to give up in case of difficulty. Also, it is important to know the parents' attitude to a child's difficulties.

2) **Making plans and setting goals** - it is crucial to the success of your work with your child that you set specific and achievable goals. If you cannot estimate (and this is difficult in the beginning) how long it will take to reach the goal, you do not need to set specific time limits. However, it's a good idea to make the goal 'small' enough that the child can see the result in a relatively short time - this will increase their motivation to keep working towards the next goal. Having specific goals helps to organize and control the learning process. It can also guide the methods and approaches to help you choose appropriate methods, some of which you will learn in the next lesson.

3) **Providing appropriate accommodations** - students with dyscalculia will benefit if they are provided with some simple tools like a calculator (they need to be taught how to use it), graph paper to help keep columns and numbers straight, reminders and alarms to help keep track of time, math apps and games to practice essential skills in a fun way (usually out of school). It would be beneficial if the school also offers some accommodations, like extra time on tests, a quiet space to work, in-school tutoring or homework assistance. All these can make a difference for a dyscalculic student.

### **Rules to follow**

To implement the strategies suggested below, it is important to follow **some simple rules**:

1) Create a calm, accepting and supportive environment - the child should see in you a person who understands his problems, whom he can trust, with whom he can share his difficulties and without worrying that he will be punished in any way for this.

2) Explain to the other students in the class what dyscalculia is and why some children find it difficult to cope with mathematics. For the student with math difficulties, a good attitude and acceptance by their classmates is of great importance to them and exceptional emotional support.

3) Adapt the assigned homework to the current skills of the student with dyscalculia - a smaller number of tasks may be more useful for him than an overwhelming number of tasks that he will either not be able to solve, or someone else (parent, sibling, or private tutor) will do for him.

4) Try to always check the homework of the student with difficulties, as well as closely monitor his work in class, so that you can notice in time if something is not understood or learned - this way you will be able to react immediately by giving additional instructions and/or explanations.

5) Always praise the student with dyscalculia for any progress made, no matter how small it may seem in the eyes of others - it is important for the child to know that his efforts lead to a result.

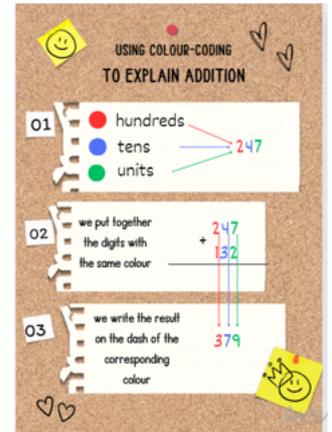
### **Strategies for supporting students with dyscalculia /Math difficulties/:**

- **Differentiated instructions – same content, lower complexity level.** This strategy requires the teacher to understand the student's needs in order to provide information of varying complexity tailored to their ability level. The language used should be simpler, the complex concepts explained using simpler words, diagrams, tables, etc. - to make them more understandable for dyscalculic learners.

- **Breaking the lesson into chunks – same content in smaller steps.** Breaking the lesson into chunks is crucial for students with difficulties. It's important for the teacher to ensure that the student understands the first concept before presenting new knowledge.

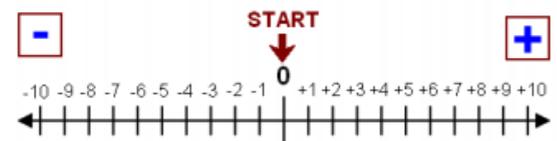
Acquiring a new concept without mastering a previous one can lead to serious confusion. In addition, as many students with DYS-difficulties have problems with following multiple instructions which require a sequential approach, try to give them instructions one by one (for example: 1/read the problem; 2/ write down what we already know; 3/ find out what we are looking for; ....). With time, the student will learn how to break the tasks himself.

- **Using colours** - the use of colours can help children with math difficulties to understand numbers, the meaning of the place of digits in a number, arithmetic operations, sequence of operations, and much more, including some complex mathematical concepts. But be careful when implementing this strategy: to avoid confusion, the same item should always be the same colour. For example, if you use blue for the tens digit in a number, it should always be blue regardless of whether it is two digits, three digits, or greater.



- **Visual Aids** - the use of visual aids is extremely beneficial for all learners, whether or not they have learning difficulties. When students can visualise mathematical concepts, it improves their understanding at all stages of learning Math. Below are some examples of how math concepts could be visualised.

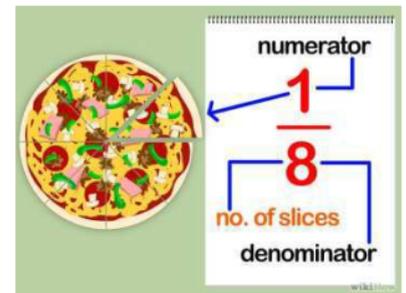
Example 1: counting ruler - helps with counting and calculation. It is visual, and it indicates the all-important zero point. This is the starting point from which, in both directions, the positive and negative numbers are positioned, starting from 0 and going to infinity.



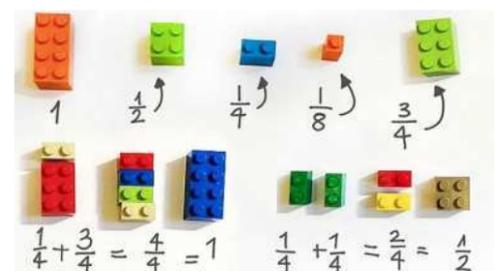
Example 2: there is an easy way to represent fractions and to understand their meaning.

1. The number above (the numerator) shows us how many parts we have
2. Next is the fraction line (which can be replaced by division)
3. The number below the fraction line (denominator) shows how many parts the whole is divided.

In this example, we have a pizza divided into 8 pieces, and we take one of them.



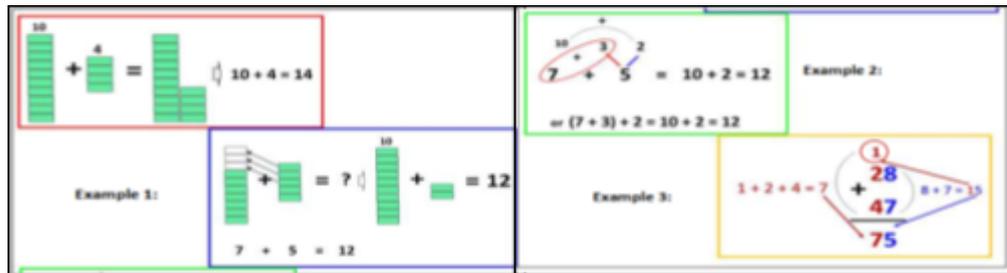
Example 3: fractions and operations with them can be explained and visualised using LEGO elements, and this is suitable for learners of all ages.



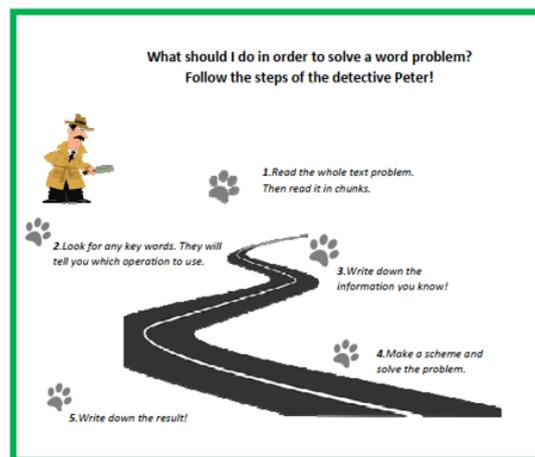
- **Using models and algorithms for solving problems** -for many students who face difficulties in learning Math, even simple mathematical operations can be a serious

challenge, they do not understand them (mostly due to deficits in the so-called mathematical thinking) and therefore cannot solve them. Having a model for performing actions and algorithms to follow makes the task clearer and easier for the child to solve.

Using a model:



Using an algorithm:



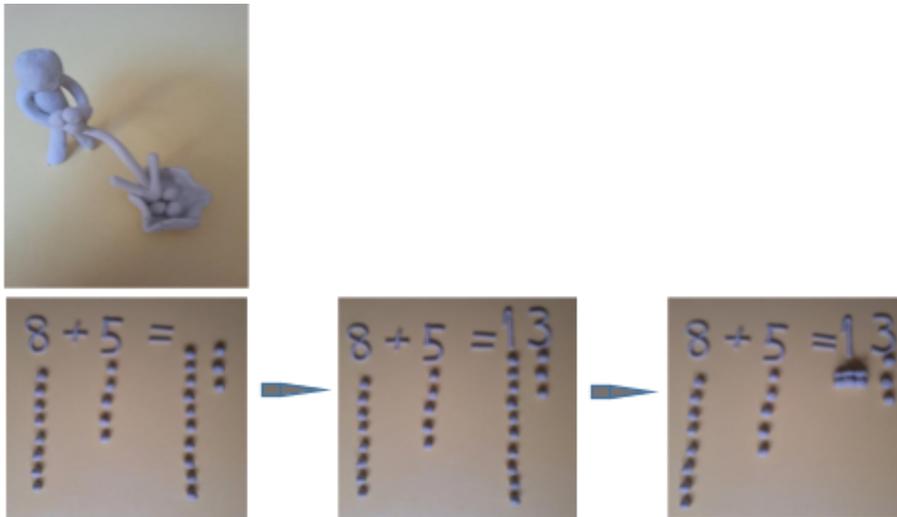
- **Link math to real life** - children find it much easier to understand mathematical concepts when they are related to and supported by real-life examples. Mathematics is all around us, and we use mathematical operations in our daily activities. Each candy we take from a box is in fact subtraction; each puzzle piece we put to build a picture is addition; if we set a table for four sets of everything (plates, forks, knives, napkins...) and it is multiplication, etc. Abstract mathematical problems can be challenging for students, but linking them to everyday activities makes it easier to visualise the situation and understand what is being asked.

- **Frequent quick review** - the automation of knowledge and skills requires significantly more time and repetition for students with learning difficulties. Take 5 minutes at the beginning of the lesson to briefly review the material that will be needed to understand the new material. This can be done in the form of a game or a quiz (Kahoot is a great option) in which everyone can participate, and will not only prepare students for the new lesson, but also create a more relaxed and pleasant atmosphere without unnecessary tension.

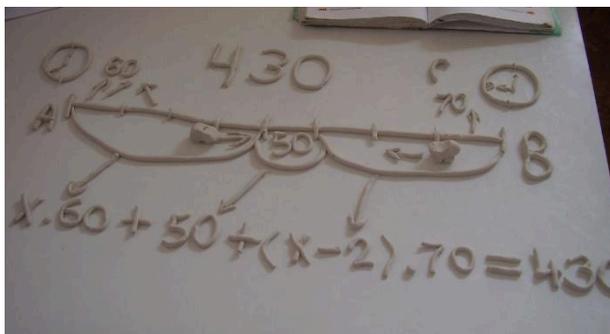
- **Hands-on Math** - learning mathematical concepts begins when children are still young. The first lessons involve recognising quantities, comparing them (less/more), associating the quantity with its representing symbol (number), etc., and typically start in kindergarten. However, numbers and figures are abstract concepts representing quantity. Young children are not yet able to think abstractly, so it's crucial, in these early stages of mathematics education, to represent abstract quantities and their operations by using real objects that children can manipulate. Children need to “see” what each concept looks like to understand it and subsequently incorporate it into their thinking process. This can be

achieved by using play dough to create tangible models of basic mathematical ideas. This approach helps children who learn best through visuals to grasp and handle abstract concepts more easily. This approach can be used for teaching practically everything – from basic numerals, arithmetical operations, geometric shapes, to very complex mathematical concepts. It can help with solving word problems as well.

Example 1: a model that represents the “ADDITION”



Example 2: /a word problem/ *The distance between cities A and B is 430 km. A car started from city A to city B at 7 am, travelling at 60 km/h. Two hours later, another car left B for A at 70 km/h. After how long will the distance between the two cars be 50 km?*



- 1/ *The first car starts from city A to city B at 7 am, at 60 km/h*
- 2/ *The second car starts from city B to city A two hours later (at 9 am), at 70 km/h*
- 3/ *X is the time we need to calculate*
- 4/ *Composing the equation*

Play dough and concept modelling is widely used in Davis Method, developed in the 1980s and used all over the world for overcoming difficulties caused by dyslexia, dyscalculia, and other specific learning difficulties ([www.dyslexia.com](http://www.dyslexia.com)).

● **Gamification** - for many students mathematics is not a favourite subject and is often labelled as “difficult”. In many cases it is associated with stress and negative emotions. One approach that has good results is the use of games in Math classes. It aims at improving the level of understanding Math concepts and operations, assuming that in a game situation it is much easier for children. On the other hand, including games in the learning process makes it more fun and enjoyable. All kinds of games could be used - video games, board or strategy games, puzzles, etc. The Internet also offers a variety of possibilities - you can prepare mathematical exercises in Quizlet, Wordwall, quizzes in

Kahoot, 3D geometric models in Tinkercad or Blippar, a virtual mathematical exhibition in Artsteps and many more.

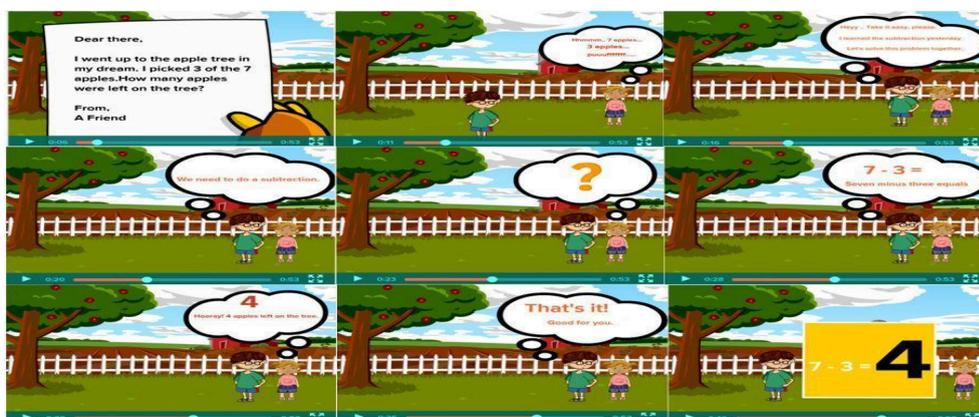
You may find some more interesting exercises, games and ideas on:

- Khan Academy ([Math | Khan Academy](#))

- Prodigy platform (<https://www.prodigygame.com/main-en/>)

- PhET (<https://phet.colorado.edu/>)

- **Story-telling** - teaching through stories can be an effective strategy for students who find it difficult to understand problems introduced to them in a numerical form or some abstract mathematical concepts. Using more descriptive or explanatory language might be easier for them to understand. With smaller children could be used toys, or well-known characters from favourite movies or computer games; to put these characters in some imaginary situation where they need to solve some equations, or word problems to continue their way. You can use some applications (e.g. BrainPop) to create such a story based on the material you need to teach and to make it more interesting for students.



Other useful links:

[IXL | Math, Language Arts, Science, Social Studies, and Spanish](#)

[AdaptedMind](#)

[The Dyscalculia Toolkit \(youtube.com\)](#)

[How to Use Touch Math - Dyscalculia - Math Help - Dyslexia Math - Touch Points \(youtube.com\)](#)

[Dyscalculia: Teaching Strategies & Modifications \(youtube.com\)](#)

[This simple game makes kids better at math \(youtube.com\)](#)

## Sources and Literature

1. Acharya, B.R.: Factors affecting difficulties in learning mathematics by mathematics learners. Int. J. Elemen. Educ. 6(2), 8-15 (2017) <https://doi.org/10.11648/J.IJEEDU.20170602.11>
2. Difficulties in Learning Mathematics: A Systematic Review, International Journal of Research and Innovation, vol.XI, iss.IX, Sept.2024
3. Davis Dyslexia Association <https://www.dyslexia.com/>

4. Fleischner, J. E., & Manheimer, M. A. (1997). *Math interventions for students with learning disabilities: Myths and realities*. *School psychology Review*, 26 (3). 397-413
5. Hamukwaya, S. T., & Haser, Ç. (2021). “It does not mean that they cannot do mathematics”: Beliefs about mathematics learning difficulties. *International Electronic Journal of Mathematics Education*, 16(1) doi:<https://doi.org/10.29333/iejme/9569>
6. Strategies and Accommodations to Support Mathematical Difficulties, Dyslexia ‘ SPELT foundation: <https://dsf.net.au/>
7. What are typical challenges students with math-related learning disabilities face? /retrieved from <https://www.washington.edu/accesscomputing/what-are-typical-challenges-students-math-related-learning-disabilities-face/>

## Chapter 3

# Applied Strategies and Implementation Guide

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Prepared by: CTA-Turkey, GEDONSOFT- Germany, LTGM - Romania

**Objective:** to provide educators and parents with step-by-step, effective, and inclusive strategies to support every student’s mathematics learning. These strategies address diverse learning styles, readiness levels, and interests to make mathematics education more accessible, enduring, and motivating.

### ***Strategy 1: Use of Visual Representations***

**Description:** presenting mathematical concepts with visual aids (shapes, graphs, concrete objects) supports students’ visual perception, making abstract concepts more understandable and memorable. Visual representations strengthen mathematical thinking processes, enable students to visualize concepts, and make learning engaging.

#### ***Step-by-step implementation:***

1) **Material collection:** students are asked to bring easily accessible objects to class that can represent mathematical concepts (e.g., fruit pictures, wooden blocks, buttons, string pieces). This increases student engagement and personalizes learning.

2) **Concept visualization:** the teacher explains a mathematical operation step-by-step using the chosen objects. For example, in addition, objects are grouped, and each step is visually demonstrated.

3) **Student participation:** students create their own visual representations. For instance, they arrange their objects or draw diagrams on paper for an addition operation. This process encourages creativity and reinforces learning.

4) **Sharing and discussion:** students present their visual models in class and explain how they work. This supports peer learning and reveals diverse perspectives.

5) **Reflection:** students are asked to write or verbally express how visual representations helped them, increasing their awareness of the learning process.

#### ***Examples:***

- **Addition:** for the operation “3 apples + 2 apples = ?”, 3 apple images and 2 apple images are drawn on the board. Each apple is paired with a number, and they are grouped to show a total of 5 apples. Students perform a similar addition operation using their chosen objects (e.g., pencils or paper pieces).

- **Fractions:** a paper circle is divided into 4 equal parts. One-fourth is shaded, and students are asked to add pieces to represent  $\frac{2}{4}$ . This helps visualize fractions.

#### ***CTA’s innovative implementation suggestions:***

##### ***Activity: Math modeling workshop***

Students model mathematical expressions using materials like Legos, colored cards, straws, or beads. For example, to show  $4 + 3$ , 4 blue Legos and 3 red Legos are combined.

- **Implementation:** students create a “Math Art Gallery,” displaying their visual models and evaluating their peers’ models.

- **Additional activity:** students write a “Math Story” using their models (e.g., “4 birds and 3 birds landed on a tree, how many birds in total?”).

- **Technology integration:** platforms like Canva, Khan Academy, Matific, or GeoGebra are used to digitize visual representations. For example, students design an addition diagram in Canva or play interactive fraction games on Matific.

- **Implementation:** create a “Digital Math Board” in class where students upload their digital visuals and receive comments from classmates.

- **Suggestion:** teachers assign a weekly “Digital Math Task” (e.g., creating an animated fraction diagram).

- **Real-world connection:** students are asked to find mathematical visuals in their environment (e.g., slicing a cake, quarter segments of a clock). These visuals are displayed on a “Math in Life” board in class.

- **Implementation:** families conduct a “Math Photo Hunt” at home with their children and send visuals to class.

- **Group work:** students work in small groups to create a collaborative project visualizing a math concept (e.g., area calculation) using square cardboards to represent a rectangle’s area.

***Differentiation tips:***

- **Visual learners:** use colorful diagrams, flowcharts, and infographics. For example, represent each number in an addition operation with a different colored circle.

- **Auditory learners:** incorporate storytelling or songs while explaining visual representations. For example, teach a “Fraction Song” (e.g., “A whole, split into two, becomes a half!”).

- **Kinesthetic learners:** allow students to physically manipulate objects. For example, cut a string to show fractions or string beads for addition.

- **Advanced learners:** encourage creating more complex visual representations (e.g., graphing an algebraic equation).

- **Students needing extra support:** start with simple, low-step visual models and provide step-by-step guidance.

## ***Strategy 2: Prior Knowledge Activation***

***Description:*** facilitating the transition to new mathematical topics by recalling students’ prior experiences and known concepts makes learning more meaningful and contextual. This strategy helps students see the connection between mathematics and daily life, boosts their confidence, and encourages active participation.

***Step-by-step implementation:***

- 1) **Pre-assessment questions:** before starting a new topic, ask open-ended questions to uncover students’ existing knowledge (e.g., “Where do we use multiplication?”). This activates their thinking.

- 2) **Mind mapping:** write students’ responses on the board to create a mind map linking prior knowledge to the new topic, structuring the learning process.

- 3) **Real-life connection:** introduce the new topic with familiar scenarios (e.g., shopping, cooking, sports), making abstract concepts concrete.

- 4) **Applied exercises:** reinforce the new topic with practical examples using students’ prior knowledge. The teacher provides guided support.

- 5) **Reflection and discussion:** ask students to explain how the new topic connects to their prior knowledge, deepening learning.

***Examples:***

- **Multiplication:** introduce multiplication with the question, “If one orange costs 4 TL, how much do 3 oranges cost?” Students share a memory of using multiplication in shopping. On the board,  $3 \times 4 = 12$  is visualized (e.g., drawing 3 groups of 4 oranges).

- **Fractions:** discuss fractions with, “If 4 friends equally share a pizza, how much does each get?” Students share experiences of sharing food, and the concept of  $\frac{1}{4}$  is introduced.

### **CTA's innovative implementation suggestions:**

#### ***Activity: real-life Math stories***

Ask students to bring an example of math used in daily life (e.g., a grocery receipt, bus ticket, recipe). Turn these into classroom problems. For example, if a receipt shows 2 kg of apples for 10 TL, calculate the price per kg ( $10 \div 2 = 5$  TL).

- **Implementation:** students keep a weekly “Math Journal,” writing and solving a real-life example each week to develop mathematical thinking skills.
- **Additional activity:** organize a “Math Market” in class where students create and solve problems using receipts or scenarios.
- **Scenario-based learning:** support math teaching with scenarios like shopping, time planning, or distance estimation. For example, “If a car travels 60 km/h, how long does it take to cover 120 km?” helps students understand speed and time.
- **Implementation:** play a “Math Adventure” game in class where students calculate distance, time, and costs in a travel scenario.
- **Suggestion:** families create a “Home Math Scenario” with their children (e.g., budgeting for a picnic).
- **Peer teaching:** students with strong prior knowledge take on a guiding role. For example, a student proficient in addition teaches peers to solve a shopping problem.
- **Implementation:** start a “Math Mentor” program in class where different students explain a topic to peers each week.
- **Benefit:** this boosts the confidence of leading students while creating a more relatable learning environment for others.
- **Community connections:** ask students to investigate how math is used in their community (e.g., price calculations at a market), embedding math in a social context.

#### **Differentiation tips:**

- **Visual learners:** use mind maps or flowcharts to connect prior knowledge. For example, show the link between multiplication and addition with a diagram.
- **Auditory learners:** encourage students to share experiences through discussions or storytelling, such as recounting a shopping memory.
- **Kinesthetic learners:** organize role-playing activities (e.g., enacting a market scene) where students solve problems by physically moving objects.
- **Advanced learners:** encourage working with more complex scenarios (e.g., a shopping problem involving discounts and taxes).
- **Students needing extra support:** start with simple, familiar scenarios (e.g., a shopping problem involving only addition).

#### ***Strategy 3: Explicit Instruction Method***

**Description:** teaching complex mathematical operations by breaking them into small, manageable steps reduces students’ cognitive load and structures the learning process. Clear instructions ensure students understand and correctly apply each step, boosting their confidence.

#### **Step-by-step implementation:**

1) **Problem presentation:** clearly write the problem (on the board, paper, or digital platform) to focus students’ attention.

2) **Step breakdown:** divide the solution process into small, logical steps. Explain each step verbally and support it visually (e.g., with a diagram or object).

3) **Guided practice:** guide students through applying each step individually. Correct errors immediately and reinforce correct steps.

4) **Independent practice:** ask students to solve a similar problem independently, with teacher support as needed.

5) **Evaluation and feedback:** review students' solutions, identify errors in specific steps, and provide individual feedback.

**Examples:**

- **Fraction simplification:** simplifying  $18/24$ :
  - Step 1: find the common divisor (6).
  - Step 2: divide the numerator ( $18 \div 6 = 3$ ).
  - Step 3: divide the denominator ( $24 \div 6 = 4$ ).
  - Step 4: write the result ( $3/4$ ).

Each step is visualized on the board with a circle diagram (a circle divided into 24 equal parts, 18 shaded, then simplified by dividing by 6).

- **Equation solving:** solving  $2x + 3 = 7$ :
  - Step 1: subtract 3 from both sides ( $2x = 4$ ).
  - Step 2: divide both sides by 2 to find  $x$  ( $x = 2$ ).
  - Step 3: check the result ( $2x + 3 = 7$ ).

Each step is explained with a balance scale visual.

**CTA's innovative implementation suggestions:**

***Activity: step cards game***

Write each solution step on a separate card. Students arrange the cards in the correct order and solve the problem. For example, for fraction simplification, cards include: "Find the common divisor," "Divide the numerator," "Divide the denominator," "Write the result."

- **Implementation:** organize a "Step Cards Race" in class where groups compete to arrange steps quickly and correctly.

- **Additional activity:** students design their own step cards and apply them to another problem.

- **Digital step tracking:** students select steps in the correct order on a tablet or phone app. For example, GeoGebra or Desmos shows equation solutions with animated steps.

- **Implementation:** organize a "Digital Math Journey" in class where students advance to the next level with each correct step.

- **Suggestion:** teachers assign a weekly "Digital Step Task" (e.g., solving a fraction problem step-by-step).

- **Gamification:** turn solution steps into a "Math Adventure" game where each correct step earns a point or "treasure." For example, a word problem is presented as steps on a treasure map.

- **Implementation:** use platforms like Prodigy where students solve step-by-step problems and earn rewards.

- **Additional activity:** students design their own math games and play them in class.

- **Peer assessment:** students check each other's solution steps and provide feedback, fostering collaboration and critical thinking.

### **Differentiation tips:**

- **Visual learners:** use color-coded diagrams or flowcharts for each step. For example, highlight fraction simplification steps in different colors.
- **Auditory learners:** use rhythmic language or audio instructions for steps. For example, teach a chant like “Divide, simplify, write the result!”
- **Kinesthetic learners:** integrate steps with physical movement. For example, students step on mats representing numbers (e.g., stepping 6 times to divide 18 by 3).
- **Advanced learners:** encourage working on more complex problems (e.g., multi-step algebraic equations).
- **Students needing extra support:** break steps into smaller parts and provide concrete examples for each.

**General differentiation tips:** lesson plans should be tailored to students’ readiness levels, learning styles, and individual characteristics to maximize their potential.

- **Content differentiation:** offer problems suited to students’ levels. For example, basic-level students solve simple addition problems, while advanced students tackle multi-step word problems.
- **Process differentiation:** adapt the learning process to students’ needs. For example, add diagrams for visual learners or physical activities for kinesthetic learners.
- **Outcome differentiation:** allow students to express knowledge in different ways. For example, one student draws a problem’s solution, while another explains it verbally.
- **Support tools:** provide manipulatives (e.g., number rods), digital tools, or guided worksheets for students needing extra support.

### **Sample lesson plan structure:**

- 1) **Objective:** ensure students understand a specific math concept (e.g., fractions).
- 2) **Introduction:** introduce the topic with prior knowledge activation (e.g., a pizza-sharing scenario).
- 3) **Main activity:** teach the concept using visual representations and explicit instructions (e.g., modeling fractions with paper circles).
- 4) **Application:** students solve the problem individually or in groups.
- 5) **Evaluation:** students present solutions and receive feedback.
- 6) **Homework:** assign a family-involved task (e.g., measuring fractions in a recipe at home).

**Conclusion:** the CTA - Turkey initiative, aligned with the vision of *Empowering Mathematics Education for All*, equips educators and parents with powerful tools to transform mathematics learning. The strategies of Visual Representations, Prior Knowledge Activation, and Explicit Instruction address diverse learning needs, fostering mathematical thinking skills. Supported by innovative applications (e.g., digital tools, gamification, real-life connections) and differentiation tips, these strategies ensure every student succeeds and remains motivated in mathematics. Collaboration among educators, parents, and students amplifies the impact of these strategies, transforming mathematics education into an inclusive, meaningful, and inspiring experience.

## Sources and Literature

1. Boaler, J. (2015). *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching*. Jossey-Bass.
2. National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. NCTM.
3. OECD. (2020). *Teachers and Leaders in Vocational Education and Training*. OECD Publishing. <https://doi.org/10.1787/59d4fbb1-en>
4. Polya, G. (1957). *How to Solve It: A New Aspect of Mathematical Method*. Princeton University Press.
5. Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2018). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Education.

Based on the practice the teachers of LTGM have, here are some example developments tailored specifically for teaching mathematics to primary school students with Special Educational Needs (SEN). These developments emphasize structure, multisensory engagement, social interaction, and differentiation, which are all key for inclusive education.

### **Development 1: “My Math Toolbox” – Personalized Manipulatives Kit**

The objective is to support tactile and visual learners through consistent, hands-on tools. There are several implementation steps:

- Each student with SEN receives a personalized “Math Toolbox” that contains number cards, fraction circles, counters, string, and beads, Velcro shapes and pattern blocks.
- During lessons (e.g., visualizing addition or fractions), students use their toolbox to mirror teacher demonstrations.
- SEN students can color-code items (e.g., red counters for units, blue for tens).
- The teacher can include pictorial instruction cards showing each operation visually (addition, subtraction, etc.).

This approach works because it offers predictable and familiar tools that reduce anxiety, support independence, and reinforce conceptual understanding through multisensory input.

### **Development 2: “Math Moves!” – Kinesthetic Stations for Concept Reinforcement**

The objective is to enhance engagement and retention through movement. The teacher has to follow a simple activity setup which includes rotating students (individually or in pairs) through 3–4 stations:

- Step-on-the-solution: use floor number mats. For “ $5 + 3$ ”, students physically step from 5 forward 3 steps.
- Fraction puzzle walk: use pizza or shape cut-outs. Students assemble correct fraction amounts by walking to parts and forming wholes.
- Math toss: throw beanbags into numbered baskets for skip counting or number bonds.

- Measure me: students measure real classroom items with string, using “estimate and check.”

The SEN support this brings to teaching is that it provides clear station visuals, uses partner support or aides for directions, and includes simplified tasks and allows extra time. This exercise works because movement helps kinesthetic learners stay focused, and repetitive physical actions reinforce number concepts and spatial reasoning.

### **Development 3: “Math & Me Stories” – Narrative-Based Math Tasks**

Their objective is to support language processing and memory through storytelling. The implementation stages are as follows:

- The teacher introduces math concepts via short, relatable stories involving students' names and their favorite things (e.g., “Ben the Builder stacks 3 blue blocks and 2 red blocks...”).
- Students create their own short math stories using visual supports or templates.
- They can use storyboards or picture sequences to guide narrative construction for addition, subtraction, or basic multiplication.

It supports SEN students because it uses visual icons or symbols (e.g., PCS or Pictures Communication Symbols, or Wigit symbols), it allows verbal or pictorial responses, and uses sentence frames (e.g., “I had \_\_\_ apples. I got \_\_\_ more. Now I have \_\_\_.”).

This development works because stories enhance comprehension and allow for personalized, meaningful connections to math operations.

### **Development 4: “Digital Buddy Math” – Assistive Tech for Individualized Practice**

The objective of this practice is to reinforce learning through interactive technology tailored to each student. The teacher can use several tools and do different activities.

- Use different sites, such as <https://www.matific.com/ro/ro/home/> , <https://www.learningblocks.tv/numberblocks/home> , or <https://learn.khanacademy.org/khan-academy-kids/> for engaging, SEN-friendly platforms.
- Assign short, leveled activities based on current topics (e.g., number bonds, basic geometry).
- Use apps with built-in scaffolding such as audio instructions and visual cues.

In order to be used in class the teacher can provide noise-canceling headphones or quiet corners for focused learning, or pair students for digital buddy activities to promote collaboration. This strategy works as technology provides immediate feedback, adjustable difficulty, and multisensory input, supporting both remediation and enrichment.

### **Development 5: “Step Cards & Visual Timelines” – Scaffolded Problem Solving**

The objective is to break down complex problems into manageable steps using visual sequencing. The application steps are available for any teacher. For concepts like simplifying fractions or solving equations, they create laminated Step Cards with simple symbols or images for each step. Then students match cards to problems on the board or in workbooks. In the classroom, the teacher can have a problem-solving timeline poster where each step is represented by a visual and a tactile action (e.g., Velcro symbols placed in order).

The support needed for SEN pupils contains color-code cards. It also includes verbal prompts with symbols and uses partner check-ins to promote peer learning. This type of strategy is efficient because chunking complex tasks into small, sequenced steps reduces cognitive load and builds procedural confidence.

### **Development 6: “What I Already Know” Boards – Prior Knowledge Activation for SEN**

The objective is to help SEN students connect new topics with familiar contexts in order to reduce learning barriers. The activity has to take place according to a plan. Before starting a topic, the teacher asks guiding questions using pictures or physical prompts (e.g., toy food for multiplication in shopping). They can display a “What We Know” board with student drawings, photos, or sentence strips. Students revisit this board during the unit to reinforce connections.

The support needed for SEN pupils includes the fact that it uses picture prompts or tactile objects to elicit ideas. It allows students to point or choose images instead of verbal responses and provides repetition and reinforcement through daily warm-ups. This strategy works as it builds confidence, promotes retrieval, and validates prior experiences.

**Differentiation for SEN** should include the following:

- Use consistent visual structure: e.g., color-coded charts, daily visual schedules, and math word walls.
- Minimize language complexity: simplify instructions, repeat key terms, and support with visuals.
- Provide choice: allow students to choose between drawing, using manipulatives, or explaining verbally.
- Involve families: assign short, inclusive home tasks (e.g., “Find 3 circles in your kitchen”) to bridge classroom and home environments.

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## **Sources and Literature**

1. Bruner, J. (1990). *Acts of meaning*. Harvard University Press.
2. Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105–117). Teachers College Press.
3. Epstein, J. L. (2011). *School, family, and community partnerships: Preparing educators and improving schools* (2nd ed.). Routledge.
4. Hall, T. E., Meyer, A., & Rose, D. H. (2012). *Universal design for learning in the classroom: Practical applications*. Guilford Press.
5. Sousa, D. A. (2016). *How the brain learns mathematics* (2nd ed.). Corwin Press.

## Chapter 4

# Emotional Well-being and Positive Learning Environment

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Developed by: **LSpecPA- LATVIA**

## **Student confidence and motivation begin with the creation of positive relationships and a supportive environment.**

When a teacher offers encouragement, students feel safer and understand that mistakes are a natural part of the learning process. As trust is built, learners feel understood and valued, which increases their willingness to try — and this directly impacts achievement in any subject.

Praising each step of the learning process (e.g., “You’ve really put effort into this!”) helps to develop a **growth mindset** and fosters **intrinsic motivation** — the desire to learn for one's own sake, not just for a grade.

One essential element of a supportive learning environment is the **use of differentiated instruction**, meaning that tasks are tailored to various skill levels. Goals and assignments become achievable when students feel capable. When learners see their own progress and set personal goals (such as “This week I’ll learn the multiplication table of seven”), the connection between knowledge and real life becomes stronger.

**Confidence and motivation are not possible without feedback** — it must always be specific, clear, and focused on improvement (not just “good job!” or “needs more work”). Students need to know what they are doing well and how they can grow — without feeling judged.

A student’s **emotional environment** is one of the most important factors influencing their well-being, academic success, and social development. When a learner feels emotionally safe, accepted, and supported, they are much better able to learn, collaborate, and grow as individuals.

### **How to create a positive emotional environment for students**

Core approaches and suggestions for building understanding and empathy toward students’ emotions, creating a safe space where they feel accepted and encouraged to express their feelings:

#### **1) Normalize mistakes as part of learning**

Say it out loud: making mistakes is normal. For example: “*Mistakes show that you're thinking - that's a good sign!*”

Share examples where adults or even you as a teacher make mistakes and learn from them.

#### **2) Create a safe and supportive environment**

Avoid public criticism - give students the chance to correct mistakes quietly or with the help of a peer.

Celebrate even small achievements — it builds confidence and encourages continued effort.

#### **3) Emphasize the thinking process, not just the result**

Praise students for their reasoning, effort, and willingness to try.

For example: “*I like how you tried several strategies to solve the problem!*”

#### **4) Personalize goals and track individual progress**

Help students set small, achievable goals - for example: “*This week I want to understand division with remainders.*” Show their progress over time by comparing their current knowledge to what they knew a month ago - not to other students.

#### **5) Include games and creative activities**

Games reduce pressure and make learning more enjoyable.

Students who fear making mistakes feel safer when tasks are presented in a playful or relaxed format.

#### 6) **Create space for emotional dialogue**

Talk about how students feel when learning math — whether it’s worry, joy, or pride.

Use emotion cards or quick daily check-ins like: *“How are you feeling about math today?”*

#### 7) **Model confidence yourself**

Show students that you believe in their abilities: *“I know you can do this — you’ve done something similar before!”*

A teacher’s positive tone can be a powerful form of support.

#### **Positive Feedback**

During the learning process, a teacher’s role is not only to correct mistakes or stop inappropriate behavior, but also to notice and reinforce positive actions: praising correct answers, appropriate behavior, positive emotional responses, and honest efforts.

It is essential for the teacher to provide **immediate feedback** on what students are doing. Student motivation and engagement can be significantly improved by commenting on their learning progress. This should be done constructively, using simple strategies and remembering that such feedback acts as a **trigger for developing student activity**.

Without feedback, there will be no progress in learning (Ruskule, 2022).

**Ruskule recommends using various strategies for providing positive feedback**, where not only the teacher gives feedback to students, but students also provide feedback to each other. These strategies include:

- idea generation;
- voting;
- developing understanding;
- monitoring the learning process;
- correcting mistakes;
- knowledge checks;
- summarizing knowledge;
- creative application of knowledge;
- peer assessment;
- self-assessment (Ruskule, 2022).

#### **A positive social-emotional environment**

Schools implement social-emotional learning (SEL) as a process through which students develop the skills to understand themselves (recognizing and managing their own emotions, thoughts, and behaviors), understand others (showing empathy and taking others’ perspectives), build positive relationships with peers and adults, and make responsible decisions.

All of these skills form the foundation for purposeful learning and reduce the risk of becoming disengaged from school. For more than two decades, the effectiveness of this approach has been demonstrated worldwide. In schools where SEL is actively practiced, students show higher academic achievement, fewer behavioral and mental health issues, and contribute to a more positive school climate. In the long term, these young people are more successful in life.

Importantly, these skills should be intentionally developed from an early age (Skola2030, 2020).

### **Well-being and psycho-emotional health**

Well-being, or good psycho-emotional health, is a dynamic state in which individuals can realize their potential, develop abilities, strengthen skills, and cope with everyday stress.

In a school context, well-being means:

- active and meaningful participation in academic and social activities;
- a strong sense of positive identity, the ability to manage thoughts and emotions, self-esteem, personal effectiveness, and a sense of autonomy;
- forming and maintaining positive and supportive relationships with teachers and peers;
- feeling safe, valued, and respected;
- experiencing a sense of belonging to one's class and school (EEA, 2024).

### **Education and the social-emotional environment in schools**

Education is not only about teaching knowledge and skills - it is also about creating a safe and supportive environment where students can learn and grow as individuals. Schools are not just places for academic learning; they are also vital spaces for the social and emotional development of young people. Children and adolescents spend a significant part of their lives at school, and this gives schools a crucial role in promoting and protecting their well-being.

Students who feel safe, supported, and connected to their school environment are more likely to achieve academic success and personal growth. They engage more actively in learning, build healthy relationships with peers and teachers, and are better prepared to face life's challenges and take advantage of future opportunities.

Students' academic achievement is strongly influenced by the social-emotional environment during the learning process. A positive classroom climate is one of the key conditions for successful learning in mathematics - it fosters curiosity, interest, and active student participation.

Creating a supportive social-emotional environment is a direct responsibility of the teacher. The teacher plans and leads the learning process, selects appropriate methods, determines how to engage students, and chooses the right types of tasks and activities.

For students with learning difficulties, it is particularly important to be informed about what will happen in the lesson - what will be done at the beginning, what follows, what types of tasks and activities are expected. A well-structured and predictable math lesson helps reduce stress, encourages a positive attitude towards learning, and supports the development and mastery of mathematical skills.

**One effective way to ensure a positive social-emotional environment and encourage active student engagement in math lessons is through play.** Games offer an engaging and enjoyable approach to learning and reinforcing fundamental math skills. They help to consolidate and deepen understanding — especially for students with learning difficulties — by making learning more accessible, interactive, and motivating.

**Example: a game for strengthening mathematical skills – “Numbers and Operations”** (See Game Board in Figure 2; see descriptions of color-coded actions in Figure 1)

**Objective:** to reinforce the ability to perform mental calculations.

**Tasks:** strengthen the understanding of the concepts *sum*, *difference*, *product*, and *quotient*; develop cooperation skills; foster patience and turn-taking.

**Materials needed:** a dice, player pieces for moving across the board (e.g. bottle caps, LEGO bricks, etc.)

**Rules and description:** the game is designed for 2 to 6 players. Each participant receives number cards. The teacher poses a mental math problem (e.g.,  $2 \times 5$ ;  $81 - 4$ ;  $90 \div 3$ ;  $43 + 15$ , etc.). The player who first raises the correct number card (to match the solution) gets to start the game. Players choose tokens to represent themselves on the board. The first player rolls the die. If they roll a 1 or 6, they may roll again after completing their turn. If a player lands on a colored square, they must solve a mental math task that corresponds to the operation indicated by the color. The game continues from there. The winner is the first player to reach 100.

**Benefits:** through gameplay, students enhance their collaboration skills, learn to respond appropriately to both success and setbacks, and build patience. At the same time, their mathematical abilities are refined and strengthened in an engaging and supportive environment.

*Figure 1. Descriptions of Actions Corresponding to Colored Square*

100	99	98	97	96	95	94	93	92	91
81	82	83	84	85	86	87	88	89	90
80	79	78	77	76	75	74	73	72	71
61	62	63	64	65	66	67	68	69	70
60	59	58	57	56	55	54	53	52	51
41	42	43	44	45	46	47	48	49	50
40	39	38	37	36	35	34	33	32	31
21	22	23	24	25	26	27	28	29	30
20	19	18	17	16	15	14	13	12	11
1	2	3	4	5	6	7	8	9	10

*Figure 2. Game Board*

	Divide the given number by 3 and move to the quotient.
	Subtract 10 from the given number and move to the difference.
	Multiply the given number by 5 and move to the product.
	Add 2 to the given number and move to the sum.
	Skip one turn.

## Chapter 5

### Case Studies and Real-World Examples

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Developed by: **CTA - TURKEY, DABG - BULGARIA, GEDONSOFT - GERMANY**

## CASE 1 – learning fractions with visual representations

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 **Observation:** a 7th-grade student struggles with adding or subtracting fractions, confusing denominator equalization. For example, they incorrectly calculate  $1/2 + 1/4$  as  $2/6$ . They find verbal explanations challenging and cannot mentally visualize fractions.

 **Goal:** help the student understand fractions through concrete and visual methods, correct misconceptions, and achieve accurate and quick results in fraction operations.

 **Classroom application:** “Fraction Workshop with Legos and Paper Circles”

### Steps:

- 1) Distribute Lego pieces of varying lengths to students. Each color represents a fraction.
- 2) Cut circles from colored paper and divide them into equal parts (halves, quarters, eighths, etc.).
- 3) Students model fractions by folding and matching parts of a whole circle (e.g., half, quarter).
- 4) Perform “ $1/2 + 1/4$ ” physically with concrete models, then symbolize it.

**Additional activity:** “Fraction Pizza Workshop” – Prepare pizza slices from cardboard. Students place slices according to orders (e.g., “What does  $2/4 + 1/4$  make?”).

 **Family involvement:** “Kitchen Fractions at Home”

### Activity suggestions:

- Halve a cookie recipe; calculate fractional measurements of ingredients.
- Use water glasses to demonstrate filling  $1/2$  or  $1/4$  of a full glass.
- Discuss fractions by cutting fruits (apples, oranges) at home.

### Evaluation:

- Have the student perform operations with visual cards and count correct answers.
- Compare pre- and post-test results (e.g., from 45% to 80% success).
- The student draws and narrates their own fraction story (e.g., “My Half-Cake Story”).

## CASE 2 – problem solving with prior knowledge activation

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 **Observation:** the student confuses the order of operations in multi-step problems, struggling to decide whether addition or multiplication is needed. They are reluctant to solve paragraph-style math problems.

 **Goal:** enable the student to apply operations in the correct order by relating them to daily life, increasing confidence in problem-solving.

 **Classroom application:** “Shopping Scenario with Problem Steps”

### Implementation process:

- Set up a market environment (toy products with price tags, paper money).
- Problem: “Ayşe bought 3 apples ( $3 \times 5$  TL) and received a 1 TL discount. How much did she pay?”
- The student realizes through the scenario that multiplication comes before subtraction.

### Add-ons:

- Students write their own shopping scenarios.
- Students take turns as cashiers, creating problems.

 **Family involvement:** “Home Market Game”

### Activities:

- Label household items; the child “shops” from the parent.
- Parents ask for change calculations, observing if the operation order is correct.

### Evaluation:

- Measure the student’s correct application of operation order (e.g., from 40% to 80%).
- Note errors in specific steps during problem-solving.
- Record home application frequency using a family observation form.

## CASE 3 – Increasing Motivation in Mathematics through Gamification

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 **Observation:** the student is reluctant to participate in math class, expressing boredom. They submit incomplete homework and come unprepared for exams, remaining passive and avoiding group work.

 **Goal:** increase the student’s interest in mathematics, encourage participation, and improve homework submission rates.

 **Classroom application:** “Task-Based Math Games”

### Tools:

- Digital platforms like “Classcraft” or “Prodigy”.
- “Math Task Notebook” (can be paper-based).

### Tasks:

- “Solve math problems for 3 consecutive days – earn 5 stars.”
- “Explain a problem to a friend – earn a helpfulness badge.”

### Reward system:

- Stars grant classroom badges, homework exemptions, or desk-switching privileges.

 **Family Involvement:** “Home Task Board”

### Activities:

- Families create a mini-task board at home (e.g., 3 problems = 1 star).
- Weekly stars earn small rewards (e.g., watching a movie together, game time).

### Evaluation:

- Compare pre- and post-participation rates.
- Note if homework submission increases from 30% to 70%.
- Record emotional indicators like smiling or active participation in class.

## CASE 4 – Learning Geometry with Augmented Reality

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 **Observation:** the student struggles to recognize 3D shapes and imagine the volume of shapes like prisms or cubes, confusing differences between shapes.

 **Goal:** help the student distinguish 3D shapes through virtual experiences and internalize volume calculations.

 **Classroom application:** “AR Geometry with Merge Cube”

## Steps:

- View prisms, spheres, or pyramids on a Merge Cube using a mobile device.
- Students rotate shapes in a virtual environment for exploration.
- Complete a shape worksheet: “How many vertices does this shape have? How many faces?”

## Additional activity:

- “Geometric Shape Hunt” – Find AR-designated shapes in class and write info cards.



## Family Involvement: “Home 3D Shape Task”

### Activities:

- Collect 3D objects (milk cartons, yogurt containers, balls) with family.
- The student labels objects: “This is a rectangular prism.”



### Evaluation:

- Conceptual test success increases by 60%.
- The student prepares a shape introduction presentation on their tablet.
- Progress is shown through pre- and post-drawings.

## CASE 5 – Game-Based Approach for a Student with Attention Deficit in Mathematics



**Observation:** the student cannot focus in class, is easily distracted, leaves operations incomplete, plays with pencils or objects, and struggles to focus on the board.



**Goal:** enable the student to sustain attention longer through physical movement and gamified activities, completing operations via games.



**Classroom application:** “Math Station Game”

### Stations:

- **Number addition run:** jump on numbers and state the result.
- **Colored dice game:** roll dice and perform operations with the numbers.
- **Puzzle problem:** assemble pieces to read and solve a question.

### Rules:

- Each station lasts 5 minutes.
- A bell signals station changes.
- Stars are collected at each station.



**Family involvement:** “5-Minute Math Games at Home”

### Sample games:

- Timed operation game: “How many problems can you solve in 1 minute?”
- Hidden number hunt: numbers are hidden around the house, found via operation questions.



### Evaluation:

- Track weekly stars collected in games.
- Record student focus duration (5–10-minute intervals) via teacher observation.
- Compare problem-solving times.

## CASE 6 – Concept Matching for Bilingual Students

**Observation:** the student confuses mathematical terms in two languages, not recognizing that “fraction” and “kesir” have the same meaning.

**Goal:** enable the student to understand and use mathematical concepts in both languages by matching them.

**Classroom application:** “Bilingual Terminology Dictionary and Matching Cards”

- Write terms in both Turkish (or native language) and English with visual support.
- Play a “Find the Match” card game for term pairing.
- Pair bilingual students for collaborative work.

**Family involvement:** “Home Terminology Hunt”

- Prepare term cards in both languages with family.
- Identify math terms in TV shows or books, recording them in a notebook.

**Evaluation:**

- Conduct a bilingual term recognition test.
- Measure the frequency of correct concept use in both languages.

## CASE 7 – Multisensory approach for teaching the multiplication tables

**Observation:** a 2nd-grade student is having great difficulty learning the multiplication table. He has managed to learn by heart the table for 1 and for 10 without being aware of exactly what multiplication means. And because he finds it difficult to remember something he doesn't understand, he faces a serious obstacle. To help him, the teacher let him use a laminated sheet of paper with all the multiplication tables. At the same time she warned the parents that the material gets more difficult, involves multiplication of multi-digit numbers, and if the child doesn't learn the table well, he won't be able to cope.

**Goal:** help the student understand the meaning of multiplication and to learn the multiplication table through simple supporting tools and multisensory approach.

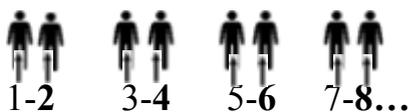
**Classroom Application:** “Fast addition of equal groups workshop”

**Steps:**

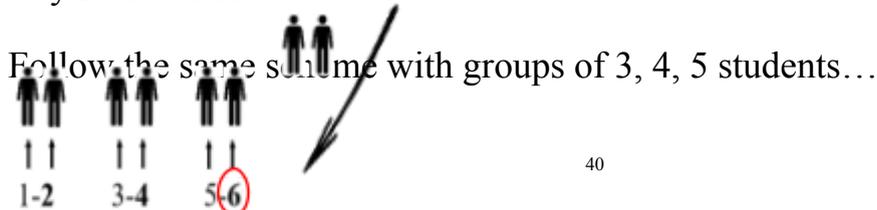
• **Ex.1:** divide the class into pairs (groups of 2). Put all children to stay in a line, with some space between the pairs.



They start counting: 1, 2, 3, 4, 5, 6..., but the first in the pair says the number quietly, the second one – loudly, so “2...4...6...” are well-heard.



The teacher asks the first three pairs to step ahead and says: “Here we have 3 groups of 2; how many students do we have?”



- **Ex. 2:** fast addition by groups. Prepare cards with the addition of equal numbers, like  $5+5+5=?$  or  $9+9+9+9=?$ , etc. Students have to calculate as fast as possible. The teacher can use a timer to register the time.

After several calculations like these, use another set of cards on which the same calculations are written, but using multiplication  $3 \times 5=?$ ;  $4 \times 9=?$ , etc. Register the time again.

Using some guiding questions, the teacher helps students conclude that using multiplication, we can calculate the sum of equal numbers much faster.

- **Ex.3:** introducing the Pythagoras multiplication table. Each student is provided with an empty grid. The teacher explains what should be written in each cell, e.g. following the row of number 6, and the column of number 6, they come together in a cell where the result of  $6 \times 6$  should be written.

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Students are asked at first to fill in the diagonal cells ( $1 \times 1$ ;  $2 \times 2$ ;  $3 \times 3 \dots$ ) – they can do it together, using addition if they don't know the answer (e.g.  $2+2=4$ ;  $3+3+3=9 \dots$ )

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

To make this activity more interesting and engaging for pupils, a toy-cars could be used. Then the explanation could be that one car drives along the row “6”, the other one along the column “6”, and at the point of the intersection (where the two cars meet) we can find the result.

	1	2	3	4	5	6	7	8	9	10
1	1									
2		4								
3			9							
4				16						
5					25					
6						36				
7							49			
8								64		
9									81	
10										100

After that students are asked to fill the first line and the first column (multiplication by 1). The teacher asks them to pay attention to the fact that  $1 \times 3 = 3 \times 1$ .

The next step is to fill the tenth line and column (multiplication by 10), and then each student has to fill the results he knows.

It is very motivating for children when they can see that there are less and less empty cells. When all multiplication tables are learnt the Pythagoras table will be completed.

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Students may still use it for some time to help themselves, but most of them very soon will feel confident enough to do multiplication without it.

### Additional activity: “Real life examples”

Once the students are familiar with the principle of multiplication the teacher can ask easy and interesting questions, like: How many legs have 6 cows, 8 chickens, 5 donkeys etc. or how many plates should be on the table if there are 7 guests and each guest needs three plates.

### Family involvement: “Board games with multiplication”

#### Activity suggestions:

- Prepare cards (might be different colours) with multiplication problems written on one side and the answer on the other. The child opens a card at random, reads the problem and has to say the answer as quickly as possible. If he doesn't remember, he turns the card over and checks. The goal is to answer as many problems as possible in a given time (e.g. 3 minutes). If you want to include movement, you can tie the answer to an action. For example,  $2 \times 3 = 6$ , jump six times, etc

- To check the multiplication tables up to 6 can be used two dice. The child rolls the dice, if one shows 4 and the other 6, they must answer how many  $4 \times 6$  is, and so on.

- There are many applications and online games that can be used for exercising. Some links:

<https://play.google.com/store/apps/details?id=coolfuture.kingofmultiplications&hl=bg>

<https://wordwall.net/bg-bg/community/%D1%82%D0%B0%D0%B1%D0%BB%D0%B8%D1%86%D0%B0%D1%82%D0%B0-%D0%B7%D0%B0-%D1%83%D0%BC%D0%BD%D0%BE%D0%B6%D0%B5>

#### Evaluation:

- Have the student independently make multiplication table cards
- Have him solve several multiplication problems (minimum 20). Let him work independently. Record the time and check the percentage of correct answers.

- After a while, do a similar check again - give the same number of problems; record the time and percentage of correct answers and compare with the previous result to register the progress.

## CASE 8 – Teaching addition and subtraction to 10

**Observation:** at the end of the 1<sup>st</sup> grade, the student cannot understand the meaning of addition and how to perform it; even when he needs to add 1, he finds it difficult and feels confused. The boy tries to avoid any activities (even games) that require using math, often complains of a headache and doesn't want to go to school on days when there are Math classes.

**Goal:** help the student learn to add numbers using visual aids and examples from everyday life. Use of the environment /at school and/or at home/ to explain the meaning of the concept of addition and to improve student's confidence in solving problems.

**Classroom application: “How many items...”**

**Steps:**

- Ex.1: counting from 1 to 10 and addition by 1**

Children usually learn to count from 1 to 10 early enough (before starting school), but many of them do it automatically without understanding the real meaning of quantity. Ask the children to count from 1 to 10 the following way:

- The first child says “one” and puts one pencil on the table; then puts below the pencil the digit 1 (it can be cut from cardboard, modelled by play-dough, or a plastic one).

|  
1

- The second child says “two” and puts another pencil on the table, and the digit 2 below it.

| |  
1 2

- Keep going until you have all 10 pencils on the table with the respective digit below each of them.

| | | | | | | | | |  
1 2 3 4 5 6 7 8 9 10

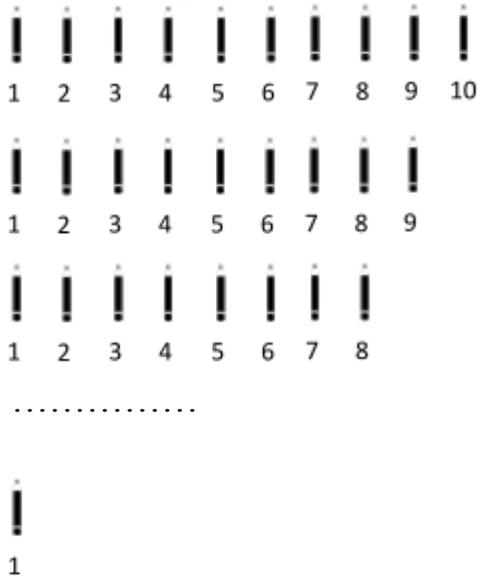
- Draw children's attention to the fact that with each pencil they add, the number of pencils on the table changes and they become MORE, because each student adds one pencil. And adding one pencil we in fact count from 1 to 10. So, adding one we come to the next number in the row.

- Children are now ready to answer a question: “If you have 5 pencils and add one more, how many pencils would you have?” Ask each child in the group one question like this, to be sure they have understood the mechanism of adding 1. At first use only questions about pencils, as they already have a mental image of pencils on the table. At some point, you can remove the pencils and keep going with similar questions: “If you have 3 apples (6 books, 9 toys...) and add one more, how many apples (books, toys...) would you have?”

- After it is understood, you can write some problems, like:  $3+1=?$ ;  $7+1=?$ ...

- Ex.2: counting backwards from 10 to 1 and subtraction by 1**

- The procedure is the same as in the Ex.1, but going backwards from 10 to 1, taking out each time one pencil.



- Draw children’s attention to the fact that each time they take out a pencil, the number of pencils on the table changes and they become LESS. And taking out pencils one by one we in fact count from 10 to 1. So, subtracting one we come to the previous number in the row.

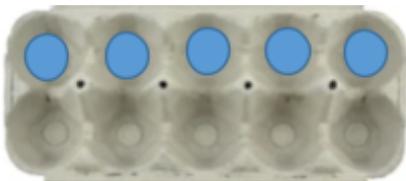
- Ask each child a question like: “If you have 6 pencils and take out one, how many pencils would you have?” At first use only questions about pencils, and go on with similar questions: “If you have 5 apples (10 books, 8 toys...) and take one out, how many apples (books, toys...) would you have?”

- After it is understood, you can write some problems, like:  $4-1=?$ ;  $9-1=?$ ...

You can use the same scheme with adding/subtracting 2, 3, ...

**NB. Move on to the next step only after you are sure that the previous step is well understood and automated.**

- Ex. 3: How many more to make it 10?



- Prepare a box with ten “cells” (you can use an egg box with ten “holes”). Let the children count the “holes”, so they are sure they are ten.

- Place a ball (or any small object) in five of the holes.

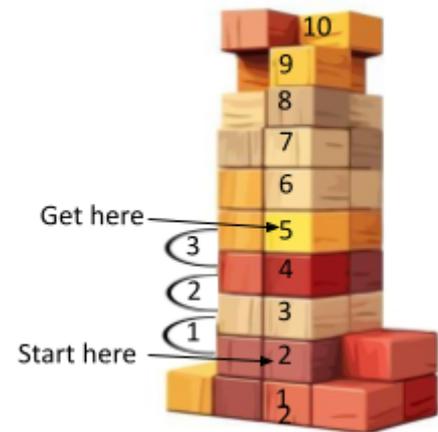
- Ask: How many more balls do you have to add to have 10? (If necessary, explain that there are still five empty “holes” which means that they need to add five more balls. Write on the board:  $5 + \rightarrow = 10$        $5 + 5 = 10$ )

- Do the same with a different number of balls/empty “holes”. Let the students use the egg box until they feel confident enough to solve the problem without visual support.

- Understanding of this operation (how many more we need to get 10) is very important before students are introduced to addition and subtraction with regrouping.

### Additional activity: “An elevator ride”

- Use Jenga blocks or Lego elements to build a 10-story building. Number the floors from 1 to 10.
- Explain to the students that the lift will move up and down, starting from a different floor and they need to calculate to which floor the lift will take them. For example: “You are on floor 2, the lift is going 3 floors up. Which floor will you reach?”, and they need to calculate  $2 + 3 = 5$ . If a student doesn't feel confident enough, he can count the floors.
- The same way the lift can go down, which will be presented by subtraction.



**Family involvement:** “Addition and subtraction in the everyday activities”

### Activity suggestions:

- Use various board games that require counting, calculating points, comparing results (e.g. Don't get angry, man). You can use domino tiles. Pay attention to the child that each tile is divided into two fields. Have the child count the tiles in the first field and write down their number, and then repeat the same with the points in the second field. Let him write down the problem and solve it. With the help of two dice, you could also compose simple addition and subtraction problems.

- When you walk with your child, add up - cars, people sitting on benches, i.e. everything you can think of.

- Use various household activities to show the child that mathematics is everywhere around us and we constantly perform various arithmetic operations, even when we are not aware of it. When giving instructions, use words that are often found in the tasks: "ADD two plates to the table." "Bring three MORE forks." etc.

- Look out the window at the parking lot in front of the apartment building. Count with your child how many cars are in the parking lot. Observe for a while. A new car arrives - this is an addition (+ 1 car; how many are there?); two cars leave - this is a subtraction (- 2 cars; how many are left?), etc.

- There are many online resources that can be used for exercising both at school and at home. Some links:

- <https://play.google.com/store/apps/details?id=com.doli.scitanie&hl=bg>

- <https://youtu.be/ebQX7uQ6Zak?si=OI7FXJriUN8JfptN>

- <https://wordwall.net/resource/24904116>

- <https://wordwall.net/resource/2844635>

- a good game for learning specific words related to addition and subtraction: <https://wordwall.net/resource/60749590>



### Evaluation:

- Have the student solve several addition and subtraction problems up to 10 (minimum 20). Let him work independently. Record the time and check the percentage of correct answers. Note what kind of mistakes he/she made.

- If necessary, explain again what you think was not well understood.
- Communicate with the family and comment on the child's behaviour at home when solving tasks.
- After a while, do a similar check again - give the same number of problems; record the time and percentage of correct answers and compare with the previous result to register the progress.

## CASE 9 – Teaching Math concepts perimeter and area

**Observation:** a student has difficulty understanding math concepts, especially the ones related to geometry. He has learnt the definition of the concepts, but this doesn't help his understanding, and without understanding, he is not able to apply the theoretical knowledge in solving math problems.

**Goal:** help the student understand the meaning of concepts like “perimeter, area, volume”, to link them to the respective formulas and to learn how to use them to solve problems through real-life examples and hands-on experience.

**Classroom application:** “Measure and calculate”

**Steps:**

- **Ex.1: Measuring length.**

Measuring is a crucial skill in geometry. Divide the class into pairs (groups of 2). Give each pair a ball of yarn.

1) Each child has to cut a rope long one handspan - when they are finished, the two children compare the lengths of rope they have cut. Conclusion: they are not quite equal because everyone's handspan is a different length. A foot is not an exact measure. To measure accurately, we use standard units of measure - metre, centimetre, millimetre, etc. (If the teacher finds it appropriate, here he can talk about converting different units of length: meters to centimetres, or centimetres to millimetres, etc.)



2) Children are asked to cut three ropes exactly the same size as a given one. The teacher observes how the children do it – if they start measuring from the beginning, or not (which can be very often seen when a child has problem with measuring), to correct and to explain/show how it should be done. Observe again to see if the child has understood and then let him to do it independently.

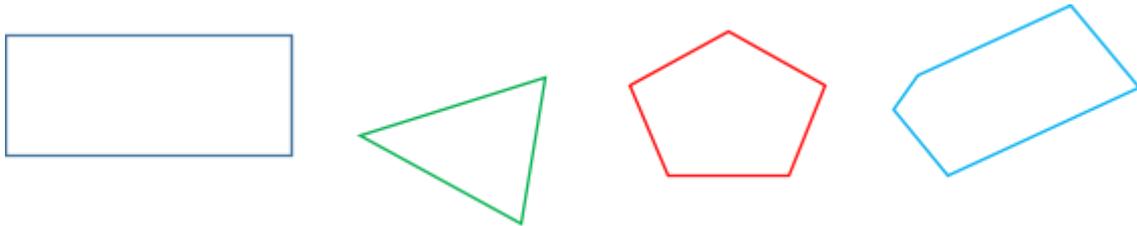


3) This time children are asked to use a ruler or tape measure to measure and cut a rope of a given length.

- **Ex. 2: Concept of perimeter.**

1) The teacher explains the concept of “perimeter” – Perimeter is the sum of the length of all sides of the figure. We find it using addition. It is the same as going around the figure, starting from one corner and walking along all sides until you get to the same corner.

2) Each child gets a piece of paper with different figures drawn. The task is to measure (using a ruler) the length of the sides of the figures and to calculate the perimeter. At the end, check the results children got and compare. Make necessary corrections, and if necessary, give additional explanations/examples.



3) Practical task: Children are asked to find the perimeter of a black(white)board, of a piece of paper, of the classroom, etc. Some more practical problems might be proposed, like “Our school yard is a rectangle 90 m wide and 130 m long. How many meters of metal fencing do we need to fence the whole yard?”

- **Ex.3: Concept of area.**

1) The teacher explains the concept of “area” – Area is the part of the plane enclosed between the sides of the figure. Depending on the figure, we use different formulas, but all of them are based on multiplication. (The teacher may explain the formulas for calculating the area of different figures, depending on the students’ grade.) It is the same as painting a wall – cover the whole wall area with paint.

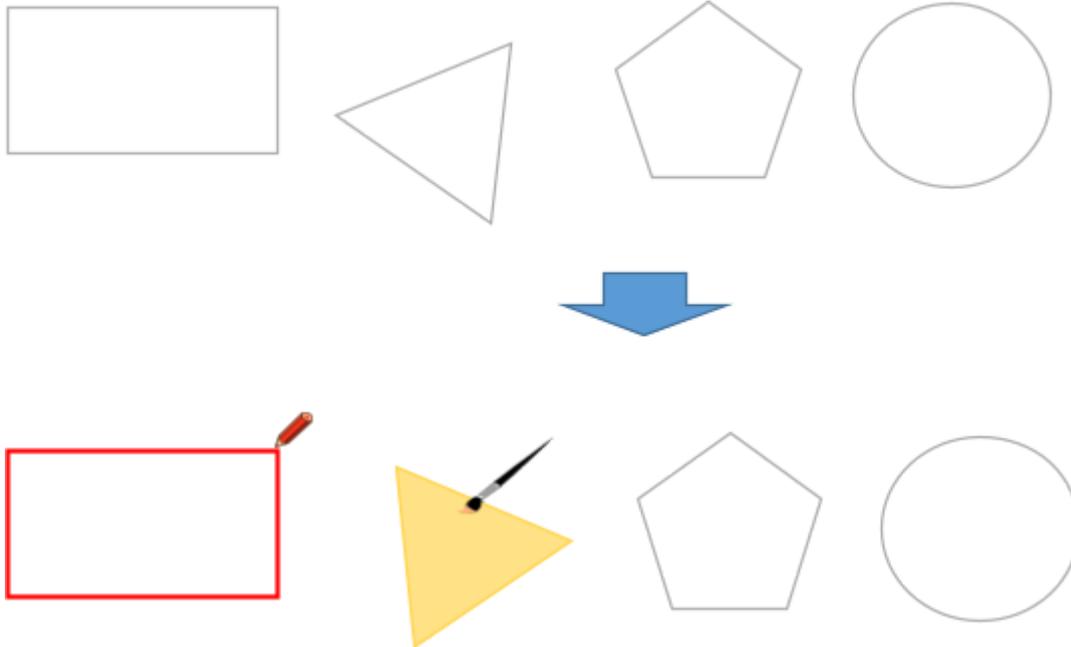


2) Each child gets a piece of paper with drawn rectangles of different sizes (length and width). The task is to measure (using a ruler) the length of the sides of each figure and to calculate its area. At the end, check the results the children got and compare. Make necessary corrections, and if necessary, give additional explanations/examples.

3) Practical task: Children are asked to find the area of the classroom walls, measuring first the length and the width of each wall, and then to calculate how much paint to buy if for one square meter they need half a kilo.

### **Additional Activity: “Perimeter or Area”**

Once the students have understood the concepts of perimeter and area, the teacher can organise a competition. The class is divided into pairs or groups of 3. Each team gets a worksheet with several different figures drawn and instructions, like: “Mark the perimeter of the rectangle. Indicate the area of the triangle...” (To mark the perimeter means to trace the figure on the outline; to indicate the area means to colour the whole figure, like in the example.)



### Family involvement: “Practical problems”

#### Activity suggestions:

- Parents can offer problems from the every-day life similar to the ones suggested above.

#### Evaluation:

- Have each student create at least two problems on finding the perimeter and the area of figures, and offer them to their classmates.
- Prepare a worksheet with 3-5 problems for measuring and calculating the perimeter and the area of different figures, and have the student solve them independently. Record the time and check the percentage of correct answers.
- After a while, do a similar check again - give the same worksheet with problems (just change the numbers); record the time and percentage of correct answers and compare with the previous result to register the progress.

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### Sources and Literature

1. Bruner, J. S. (1966). *Toward a Theory of Instruction*. Harvard University Press.
2. Boaler, J. (2016). *Mathematical Mindsets*. Jossey-Bass.
3. OECD. (2021). *Teachers and Leaders in VET*. <https://doi.org/10.1787/59d4fbb1-en>
4. Hattie, J. (2012). *Visible Learning for Teachers*. Routledge.
5. Tomlinson, C. A. (2014). *The Differentiated Classroom*. ASCD.
6. Polya, G. (1957). *How to Solve It*. Princeton University Press.
7. Van Garderen, D. (2006). “Spatial Visualization and Mathematical Problem Solving,” *Journal of Learning Disabilities*, 39(6), 496–506.

8. OECD. (2023). *Education Policy Outlook*.  
<https://www.oecd.org/education/policy-outlook/>
9. National Council of Teachers of Mathematics. (2020). *Catalyzing Change in Middle School Mathematics*.
10. Papert, S. (1993). *The Children's Machine: Rethinking School in the Age of the Computer*. Basic Books.
11. Rose, D. H., & Dalton, B. (2009). "Learning to Read in the Digital Age," *Mind, Brain, and Education*, 3(2), 74–83.

Teachers in Romania have started to pay more attention to finding different strategies of teaching math. We have tried to put together a series of practical, hands-on examples of how core mathematics concepts can be taught to primary pupils with Special Educational Needs (SEN), including those with learning difficulties, autism, dyscalculia, or speech and language needs. These strategies are designed for accessibility, engagement, and skill development.

### 1) Number sense and counting

The concept that needs to be taught is understanding numbers and quantity. The strategy the teacher can use is called Multi-sensory "Number Boxes".

**The materials needed:** shoeboxes labeled 1–10, filled with that number of small objects (e.g. 3 toy cars, 7 blocks).

**Activity:** pupils match number cards to the correct boxes.

**Adaptation:** use tactile objects for visually impaired students, or picture cards for non-verbal learners.

This works because it connects symbolic numbers to real quantities and allows visual, tactile, and verbal interaction.

### 2) Addition and subtraction:

The concept the students need to grasp is combining and taking away. The strategy that can be used by the teacher is called "Math Stories with Manipulatives".

**The teacher creates a short story:** "3 bears went into the forest. 2 more joined. How many now?"

Pupils act out the story using counting bears or toy animals.

Once the students have solved the exercises in the story, it can be extended with visual sentence frames:  $3 + 2 = ?$ , using pictures and symbols.

**Adaptations:**

- If a pupil has speech/ language needs, the teacher can use structured sentence stems.
- The teacher can provide number lines, ten-frames, or tactile counters

### 3) Place Value

The concept taught here is one of the most important ones in Mathematics, i.e. understanding tens and ones.

**Strategy: base-10 building with blocks or beads**

- Use straws, LEGO, or beads grouped in tens and single units.
- Pupils physically bundle 10 straws together to understand "ten" as a unit.
- Ask: "How many tens?" "How many ones?" then write the numeral.

When the teacher uses visual + hands-on, we are talking about concept anchoring, especially useful for dyscalculia or MLD (moderate learning difficulties).

#### 4) Time (reading clocks)

The concept of telling time to the hour and half-hour is very important at primary level. The strategy the teacher can use is the interactive clock matching game.

- Pupils match analog clock faces to digital times and “real life” cards (e.g., lunchtime = 12:00).
- Use jumbo teaching clocks they can move by hand.
- Introduction with a whole-class mimic: “Show me 3 o’clock!”

The adaptation that can be made for autistic students is to use routine-based visual schedules to link time with daily activities.

#### 5) Measurement and comparison

The concept taught in this case is comparing length or weight. The teacher can use a simple strategy, called “Measure the Classroom” Quest.

- Give students strips of paper or cubes to measure desks, books, or even their shoes.
- Use “longer/shorter”, “heavier/lighter”, “thicker/thinner” comparison language.

**The strategy can have several adaptations:**

- For motor difficulties: use fixed-length measuring sticks.
- For language needs: use picture cues or AAC (Augmentative and Alternative Communication) boards with symbols.

#### 6) Shapes and space

The concept to be taught is 2D and 3D shapes recognition. Teachers can use the strategy called “Shape Hunt” with Sensory Bins or Room Exploration.

- Pupils are asked to search for real-life shapes (e.g. “Find something round”).
- Teachers can create **tactile shape cards** or foam shapes for hands-on exploration.

As a fun extension, teachers can build 3D shapes from sticks and marshmallows, which is great for visual-spatial learners.

#### 7) Data handling

The concept the student needs to understand is sorting and charting. The strategy that can be used is called “Classroom Data Detective”.

- Pupils in the class answer a question like “What’s your favourite fruit?”.
- The teacher uses colored counters or stickers to represent each choice.
- They build a pictogram or bar graph together on a poster.

There are other possible adaptations for this strategy.

- Use Velcro or magnetic boards for fine motor accessibility.
- Use symbol-supported labels for EAL or speech-delayed learners.

#### 8) Patterns and sequences

The concept to be taught is recognizing repetition and order. The strategy used by the teachers is called “Pattern Bracelets”.

- Pupils create color or shape patterns using beads: red-blue-red-blue...
- Extend by having them copy, continue, or describe the pattern aloud.

Adaptations: use texture patterns (smooth/rough) for VI students or to build sensory awareness.

#### Best practice tips

- **Repeat and scaffold:** reinforce core skills using multiple formats (visual, verbal, physical).

- **Use visuals:** number lines, picture supports, and clear task boards help structure learning.
- **Break tasks into small steps:** use success criteria or checklists for clarity.
- **Incorporate interests:** if a child loves dinosaurs, count toy dinos or measure their “dino foot stomp” distance!
- **Celebrate success:** use token boards, praise, and visual reward charts for motivation.

## Chapter 6

### Use and Outcome of the E-Guide

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Prepared by: **CTA-TURKEY**

**Objective:** the aim is to ensure effective use of the E-Guide, which contains alternative and innovative methods for mathematics teaching, by all stakeholders (teachers, parents, students, and the educational community). This section explains step by step how to implement the E-Guide, highlights its benefits, and invites stakeholders to contribute to its improvement through collaboration, feedback, and experience sharing. Our goal as CTA is to make mathematics education accessible, inclusive, and inspiring for every student, transforming learning processes.

**Key benefits of the e-guide:** the e-guide is a user-friendly and flexible resource designed to strengthen mathematics teaching. Its benefits for students, teachers, and parents are summarized below:

- **Inclusive and accessible teaching:** the e-guide offers methods suitable for different learning styles, readiness levels, and individual needs. It provides alternative approaches especially for students with learning difficulties or low self-confidence in mathematics, helping everyone to reach their potential.

*Example:* includes diagrams for visual learners, physical activities for kinesthetic learners, and audio instructions for auditory learners.

- **Supporting teachers' professional development:** the guide allows teachers to diversify their lesson plans and integrate innovative pedagogical approaches such as game-based learning, problem-solving techniques, and group work to make lessons more dynamic and effective.

*Example:* a teacher can use the "Mathematics Station Game" from the guide to increase student attention and participation.

- **Strengthening parental involvement:** the E-Guide provides practical suggestions for parents to actively support their children's learning. Home-based activities make it easier for families to support mathematics learning and strengthen school-family cooperation.

*Example:* parents can help their children learn fractions with the "Kitchen Fractions at Home" activity.

- **Digital and audio-visual support:** the guide is enriched with digital content (videos, animations, interactive applications) and audio-visual materials that engage students, concretize abstract concepts, and make learning fun.

*Example:* digital activities integrated with platforms like Khan Academy or Matific help students learn fractions or geometric shapes interactively.

- **Academic achievement and support:** the e-guide aims to increase students' academic success and provide targeted support for those with learning difficulties, improving mathematical thinking skills and long-term learning outcomes.

*Example:* differentiation tips in the guide support students with special needs through tailored teaching methods.

- **Social and cultural connections:** the e-guide adapts mathematics teaching to local and cultural contexts to increase students' connection to the learning process. Everyday scenarios from Turkey (e.g., market shopping, recipes) are frequently used.

*Example:* an activity encourages students to practice price calculations at the neighborhood market.

## Instructions for using the e-guide:

### *For teachers:*

- Integrate guide sections into weekly lesson plans.
- Use hands-on methods like group work, games, problem-solving.
- Collect regular feedback from students.
- Integrate digital tools (e.g., Canva, GeoGebra).
- Differentiate and personalize teaching using the guide.

### *For parents:*

- Review the parent section of the guide.
- Apply simple activities at home.
- Collaborate with teachers.
- Motivate and reward children.
- Use recommended digital tools together.

### *For other stakeholders:*

- **School administrators:** integrate the guide into professional development.
- **Education technologists:** develop digital content tailored to local needs.
- **Community leaders:** promote the guide in neighborhoods and local centers.

## Implementation process and expected outcomes:

- **Initial phase:** seminars for teachers and parents, resource distribution, pilot applications.
- **Implementation phase:** classroom and home activities, feedback collection.
- **Evaluation and improvement:** analyzing results, integrating feedback, providing continuous support.

### *Expected results:*

- Students: better understanding, increased confidence, positive attitude.
- Teachers: more effective lessons, increased participation, professional growth.
- Parents: active involvement, stronger school-family cooperation.
- Community: increased awareness, support for educational initiatives.

## Conclusion

The e-guide is a powerful tool to make mathematics education accessible, inclusive, and motivating for all. It reveals students' learning potential, offers innovative methods to teachers, and actively involves parents. Enriched with digital content, audio-visual materials, and local contexts, it promotes not only academic success but also a positive attitude towards mathematics. We invite you to apply the guide, share feedback, and join the global educational community. Together, let's strengthen mathematics education for all!

# CONTRIBUTORS



At the heart of inclusive education in Latvia is **Latvian Association of Special Education Teachers (LSpecPA)** – a powerful voice for special education professionals across the country. With a community of passionate educators, speech therapists, psychologists, and researchers, the association works tirelessly to advocate for equal learning opportunities and to uplift the standards of special pedagogy. LSpecPA stands out not only for its nationwide impact but also for its dedication to empowering those who support learners with diverse needs.



Where technology meets imagination – **GedonSoft GmbH** is a trailblazing software company based in the city of Bremen. Known for crafting cutting-edge digital tools, this dynamic team specializes in 3D environments, intuitive user interfaces, and assistive technologies that make learning accessible and engaging. Whether it's immersive visuals or intelligent software, GedonSoft turns complex tech into powerful, human-centered solutions.



A beacon of hope and advocacy in Bulgaria, **SDRUZHENIE "ASOCIACIA DYSLEXIA - BULGARIA (DABG)** is a dedicated non-profit organization championing the rights and potential of learners with dyslexia and other learning challenges. For nearly two decades, it has been a trusted source of support, training, and inspiration for teachers, families, and students. With a strong network of specialists and a passion for inclusion, the association is changing the narrative around learning difficulties – one empowered learner at a time.



A hub of excellence in mathematics and innovation, **Liceul Teoretic Grigore Moisil (LTGM)** in Timisoara is more than just a school – it's a launchpad for bright futures. With over 1,800 students and a team of visionary educators, the school inspires academic curiosity, creativity, and digital fluency from an early age. Proudly ranked among the top schools in its region, it combines tradition with forward-thinking to shape confident, capable citizens of tomorrow.



Creativity. Compassion. Change. **Kreatif Düşünce ve Eğitim Derneği (Creative Thinking and Education Association - CTA)** brings these values to life in everything it does. Based in Türkiye, this energetic NGO is on a mission to transform education through inclusive, creative, and learner-centered approaches. With its inspiring blend of hands-on training, community support, and advocacy for special needs education, the association empowers educators and learners to thrive – no matter the challenge.



Join the movement of discovering  
Math teaching from the other angle.  
Scan the QR code and follow us!



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